



# Spatiotemporal modulated solitons in a quasi-one-dimensional spin-1 Bose–Einstein condensates

Fei-Yan Liu<sup>a</sup>, Su-Yong Xu<sup>b</sup>, Houria Triki<sup>c</sup>, Amitava Choudhuri<sup>d</sup>, Qin Zhou<sup>a,e,\*</sup>

<sup>a</sup> Research Group of Nonlinear Optical Science and Technology, Research Center of Nonlinear Science, School of Mathematical and Physical Sciences, Wuhan Textile University, Wuhan 430200, China

<sup>b</sup> College of Optical, Mechanical and Electrical Engineering, Zhejiang A&F University, Lin'an 311300, China

<sup>c</sup> Radiation Physics Laboratory, Department of Physics, Faculty of Sciences, Badji Mokhtar University, P.O. Box 12, 23000 Annaba, Algeria

<sup>d</sup> Department of Physics, The University of Burdwan, Golapbag 713104, West Bengal, India

<sup>e</sup> State Key Laboratory of New Textile Materials and Advanced Processing Technologies, Wuhan Textile University, Wuhan 430200, China

## ARTICLE INFO

### Keywords:

Bose–Einstein condensates  
Bright/dark solitons  
Hirota bilinear method

## ABSTRACT

In this paper, we investigate the nonautonomous bright/dark solitons in a quasi-one-dimensional spin-1 Bose–Einstein condensates through a three coupled Gross–Pitaevskii (GP) system with space–time-dependent external potential and temporally modulated gain/loss distributions. Based on the Hirota bilinear method, analytically construct the bright soliton solutions when the coupled GP system exhibits attractive interaction while we obtain the dark soliton solutions when the coupled GP system exhibits repulsive interaction. The influence of spatiotemporal modulated external potentials, such as the gain/loss distribution  $\Gamma(r)$ , bright/dark soliton dynamics is analyzed in detail via the analytical solutions. By taking different  $\Gamma(r)$ , obtain different types of bright solitons, including periodic, dromion-like and parabolic solitons, and derive dark solitons on different backgrounds, such as periodic, parabolic and kink backgrounds. We analyze the regulatory effects of different wavenumber ratios on the attraction and squeezing of bound-state solitons. Through the asymptotic analysis, we find that the interactions between two solitons are elastic. In addition, we conduct research on the forward and inverse problems of the above results via the parallel hard-constrained physical informed neural network (phPINN) method. The predicted solitons and potential functions are in good agreement with the exact solitons and potential in the system.

## 1. Introduction

Bose–Einstein condensation (BEC) represents one of the most specific and fascinating quantum phenomena in nature [1]. In essence, it is a genuinely quantum-mechanical phase transition which is driven by the particle statistics and not by their interaction. Especially, a spinor BEC is a BEC with an internal atomic spin degree of freedom displaying a rich variety of magnetic effects [2]. In such interesting multi-component condensed system, there exist several phases below the transition temperature and the phases are dependent on the nature of the interaction. It should be mentioned here that in a conventional magnetic traps, the spin degrees of freedom are frozen and the BEC is described by a scalar order-parameter. In contrast, when the BEC is trapped using an optical potential, the spin of each atom is free to evolve due to the interparticle interaction. The order-parameter describing a BEC with spin internal degrees of freedom is referred to as a spinor BEC [3–6]. Usually, spinor BECs feature an intrinsic three

component structure, which is due to the differences between different hyperfine spin states of atoms [5]. It is also to be noted that such a spinor system has been first experimentally realized in a gas of  $^{23}\text{Na}$  atoms with hyperfine spin  $F = 1$ , in an optical dipole trap [4]. Moreover, it has been reported that due to the interparticle interaction, the direction of atomic spins can change, and therefore, spinor BECs exhibit certain spin textures [7–9]. Nowadays, spinor BECs have an important role in physics because the spin degrees of freedom can generate rich quantum dynamics and abundant phenomena, including spin texture (i.e., spatial variation of the spin direction [2]), magnetic crystallization, fractional vortices [10–12]. Importantly, experimental and theoretical studies on spinor BECs have revealed various interesting phenomena, such as quantum junction [13], polarity to ferromagnetic phase transition [14], condensation excited by Magnon [15], and various nonlinear excitations composed of dark/bright solitons [16–23], rogue waves [24], soliton complexes [25], vortices [26–28], etc..

\* Corresponding author at: Research Group of Nonlinear Optical Science and Technology, Research Center of Nonlinear Science, School of Mathematical Physical Sciences, Wuhan Textile University, Wuhan 430200, China.

E-mail address: [qinzhou@whu.edu.cn](mailto:qinzhou@whu.edu.cn) (Q. Zhou).

<https://doi.org/10.1016/j.chaos.2024.114947>

Received 28 February 2024; Received in revised form 27 April 2024; Accepted 28 April 2024

Available online 3 May 2024

0960-0779/© 2024 Elsevier Ltd. All rights reserved.