THE UNIVERSITY OF BURDWAN
Physics Department
Burdwan 713 104
INDIA

Dr. Tanmoy Banerjee

Associate Professor

Phone: 0342 -2557800 (O)
Email :
tanbanrs@yahoo.co.in

Date: 24.9.21

To

The Under Secretary (FD-III)

University Grants Commission

New Delhi-110002

SUB: Final Report, Statement of Expenditure and Utilization certificate

Sir,

This is with reference to the UGC Minor project entitled "Studies on synchronization of chaos in electronic circuits and its applications" (F No: 34-506/2008 (SR)) under my principle investigation. Enclosed herewith please find the following:

(i) Final Report of the work done on the minor project.

(ii) Statement of Expenditure (audited)

(iii) Utilization Certificate (audited)

(iv) Bank mandate form

(v) Certificate from the Registrar

Thank you.

Yours sincerely,

Tanmoy Banerjee

(Tanmoy Banerjee)

UNIVERSITY GRANTS COMMISSION
BAHADUR SHAH ZAFAR MARG
NEW DELHI – 110 002.

STATEMENT OF EXPENDITURE IN RESPECT OF MINOR RESEARCH PROJECT

?1. Name of Principal Investigator DR TANMOY BANERJEE
?2. Deptt. of University/College DEPT. OF PHYSICS, BURDWAN UNIVERSITY
?3. UGC approval No. and Date F. No: 34 - 506/2008 (SR) Dated. 15 Jan 2009
?4. Title of the Research Project "Studies on synchronization of chaos in electronic circuits and its applications"
?5. Effective date of starting the project 1.2.2009
?6. (a) Period of Expenditure: From 1/2/2009to 31.1.2011
?h. Details of Expenditure:

S.No.	Item	Amount Approved Rs.	Expenditure Incurred Rs.
i.	Books & Journals	12000	**12000**
ii.	Equipment	50000	**49989**
iii.	Contingency	35000	**31912**
iv.	Field Work/Travel	NIL	**Total: 93,901**
v.	Hiring Services	NIL	
vi.	Chemicals & Glassware	NIL	
vii.	Overhead	NIL	
viii.	Any other items (Please specify)	NIL	

i. Staff **NA**

4. It is certified that the grant of **Rs 93500** (Rupees Ninety three thousand and five hundred) received from the University Grants Commission under the scheme of support for MInor Research Project entitled "Studies on synchronization of chaos in electronic circuits and its applications"vide UGC letter No. F. NO 34-506/2008 (SR) dated 15 Jan, 2009 an amount of **Rs. 93901/-** (Rupees ninety three thousand nine hundred and one) has been utilized for the purpose for which it was sanctioned in accordance with the terms and conditions laid down by the University Grants commission and an excess expenditure of **Rs.401** (Rs Four hundred and one) which is to be reimbursed to The University of Burdwan.

SIGNATURE OF THE PRINCIPAL INVESTIGATOR

Tanmoy Banerjee
Dr. T. Banerjee
Associate Professor
Department of Physics
The University of Burdwan
Burdwan-713104

21.9.21
REGISTRAR

Registrar (Officiating)
The University of Burdwan
Rajbati, Burdwan-713104
West Bengal

THE UNIVERSITY OF BURDWAN RAJBATI BURDWAN 713104

17-9-2021
STATUTORY AUDITOR

JADAB KUMAR RAY & Co. BURDWAN 53411 Chartered Accountants

ज्ञान-विज्ञान विमुक्तये

Annexure - IV

UNIVERSITY GRANTS COMMISSION
BAHADUR SHAH ZAFAR MARG
NEW DELHI – 110 002.

Utilization certificate

Certified that the grant of **Rs. 93500** (Rupees Ninety three thousand and five hundred) received from the University Grants Commission under the scheme of support for Minor Research Project entitled "Studies on synchronization of chaos in electronic circuits and its applications" vide UGC letter No. F No: 34-506/2008 (SR) Dated 15 Jan. 2009, an amount of **Rs. 93901/-** (Rupees ninety three thousand nine hundred and one) has been utilized for the purpose for which it was sanctioned and in accordance with the terms and conditions laid down by the University Grants commission and an excess expenditure of **Rs.401** (Rs Four hundred and one) which is to be reimbursed to The University of Burdwan.

21.09.21
REGISTRAR

17.9.2021
STATUTORY AUDITOR

SIGNATURE OF THE PRINCIPAL INVESTIGATOR

Tanmoy Banerjee

Dr. T. Banerjee
Associate Professor
Department of Physics
The University of Burdwan
Burdwan-713104

Registrar (Officiating)
The University of Burdwan
Rajbati, Burdwan-713104
West Bengal

Annexure -III

ज्ञान-विज्ञान विमुक्तये

UNIVERSITY GRANTS COMMISSION
BAHADUR SHAH ZAFAR MARG
NEW DELHI – 110 002.

Final Report of the work done on the Minor Research Project. (Report to be submitted within 6 weeks after completion of each year).

1. Project report No. 1st /2nd /3rd /Final: **Final**
2. UGC Reference No.: **Project no: F No: 34-506/2008 (SR)**
3. Period of report: from **2009 to 2011**
4. Title of research project: Studies on synchronization of chaos in electronic circuits and its application.
5. (a) Name of the Principal Investigator: **Dr. TANMOY BANERJEE**
   (b) Deptt. and University/College where work has progressed : **Department of Physics, Burdwan University, Burdwan.713104, West Bengal.**
6. Effective date of starting of the project: Feb 2009
7. Grant approved and expenditure incurred during the period of the report:
   a. Total amount approved Rs. :93,500/-
   b. Total expenditure Rs. :93,901/- (Excess Expenditure Rs.401/-)

Report of the work done: (Please attach a separate sheet)

(i)Brief objective of the project: **Please see Ann.1**

ii. Work done so far and results achieved and publications, if any, resulting from the work (Give details of the papers and names of the journals in which it has been published or accepted for publication: **Please seen Ann 2**

(iii) Has the progress been according to original plan of work and towards achieving the objective. if not, state reasons: **YES**

(iv) Please indicate the difficulties, if any, experienced in implementing the project: **None**

(v) If project has not been completed, please indicate the approximate time by which it is likely to be completed. A summary of the work done for the period (Annual basis) may please be sent to the Commission on a separate sheet : **NA**

vi. If the project has been completed, please enclose a summary of the findings of the study. Two bound copies of the final report of work done may also be sent to the Commission : **Please see Annex 2**

vii. Any other information which would help in evaluation of work done on the project. At the completion of the project, the first report should indicate the output, such as (a) Manpower trained (b) Ph. D. awarded **(c) Publication of results** (d) other impact, if any : **Annex 2**

Tanmoy Banerjee
SIGNATURE OF THE PRINCIPAL INVESTIGATOR
Dr. T. Banerjee
Associate Professor
Department of Physics
The University of Burdwan
Burdwan-713104

24.09.2021
SIGNATURE OF THE REGISTRAR

REGISTRAR (Officiating)
THE UNIVERSITY OF BURDWAN
BURDWAN-713104

Studies on synchronization of chaos in electronic circuits and its application.

UGC MINOR RESEARCH PROJECT

(Project no: F No: 34-506/2008 (SR))

Final report

Principal Investigator

Dr. Tanmoy Banerjee

Department of Physics

Burdwan University

Burdwan.713104

West Bengal

Final report

Ann-1
Objective:

Main objective of the present project is to

i) Design of chaotic electronic oscillators
ii) Study synchronization of chaos in those electronic circuits.
iii) Explore the potential of the study in chaotic communication system.

New chaotic electronic oscillators (SAB based chaotic oscillator) has been designed. Present work has given emphasize on the synchronization problem of chaotic SAB circuits. Further the present work search for application potentiality of the chaotic synchronization of electronic circuits in electronic communication. In the context of recent interest in secure communication system using chaotic signals, this work will be useful in both the academic and technical purpose.

Ann-2

Works completed:

1. Studies on chaotic Jerk circuit:

I have thoroughly investigated the behavior of a Jerk circuit with absolute nonlinearity. Through Numerical studies we have derived Figenbum constant, bifurcation diagram, phase plane plots. Also we have designed experimental chaotic Jerk circuit. It has been found experimentally that the circuit shows complex behavior like chaos and bifurcation. Experimental Power spectrum has been measured and it shows a broad nature, which is the characteriostic of a chaotic signal. Also Figenbum constant has been computed experimentally.

2. Design of two new chaotic circuits:

Two simple autonomous chaotic electronic circuits have been proposed in this paper. The core of each of the circuits consists of a Single Amplifier Biquad (SAB). We have proposed two configurations of converting this SAB into chaotic oscillators using suitable passive nonlinear element and a storage element in the form of an inductor. The mathematical models of the proposed chaotic circuits have been constructed, which are fourth order autonomous nonlinear differential equations. The behavior of the proposed circuits has been investigated through numerical simulations, Spice based circuit simulations and electronic hardware experiments and they agree well with each other. It has been found that both the circuits show complex behaviors like bifurcations and chaos for a certain range of circuit parameters.

3. **Hardware design of Lorenz circuit:**

I have used electronic workbench software to simulate the Lorenz circuit in its hardware level. It has been found that with varying parameter (e.g. an resistor) a Lorenz circuit shows chaos. The occurrence of butterfly attractor has been observed. Finally, in our laboratory I have designed a Lorenz circuit using op amps, multiplier chips. The circuit shows complex behavior like bifurcation and chaos.

4. **Synchronization of chaos in Jerk circuit:** To harness the richness of chaos is one of the most widely recognized research topics in academic and industrial world. Synchronization of chaos has opened a new era of chaotic electronic communication systems. Chaos based communication system is believed to be secure and interference free. In the present work, the feedback synchronization of two chaotic jerk oscillators and its application in chaotic electronic communication system have been investigated. For that, we have designed electronic jerk oscillators with absolute type nonlinearity and coupled them using one way coupling. It has been found that, for suitable coupling strengths, two oscillators can be made synchronized. Further, by exploiting this synchronization process, we have proposed a chaos based electronic communication scheme. The practical implementation of this communication scheme has been verified experimentally.

   5. Chaotic communication system

One of the applications of chaos synchronization is chaos based communication. We have investigated the possibility of chaotic communication using chaotic synchronization of two Jerk circuits. The scheme is as follows: at first add the sinusoidal modulating signal with the chaotic y-variable of Drive circuit. Two Jerk circuits are made synchronized by using feedback synchronization as discussed earlier. Now (y+modulating) signal is subtracted by v-variable of the Response Jerk circuit. If two Jerk circuits are in a synchronized state then the subtracted output is simply the modulating signal. From the security point of view, if an unauthorized user does not know the x-variable then he can not reconstruct the modulating signal. Also since the modulating signal is masked by the chaotic signal it is not possible by any user to get an idea of the modulating signal.

Results:

Following are some experimentally obtained results in the experiments stated above:

1. **Jerk circuit**

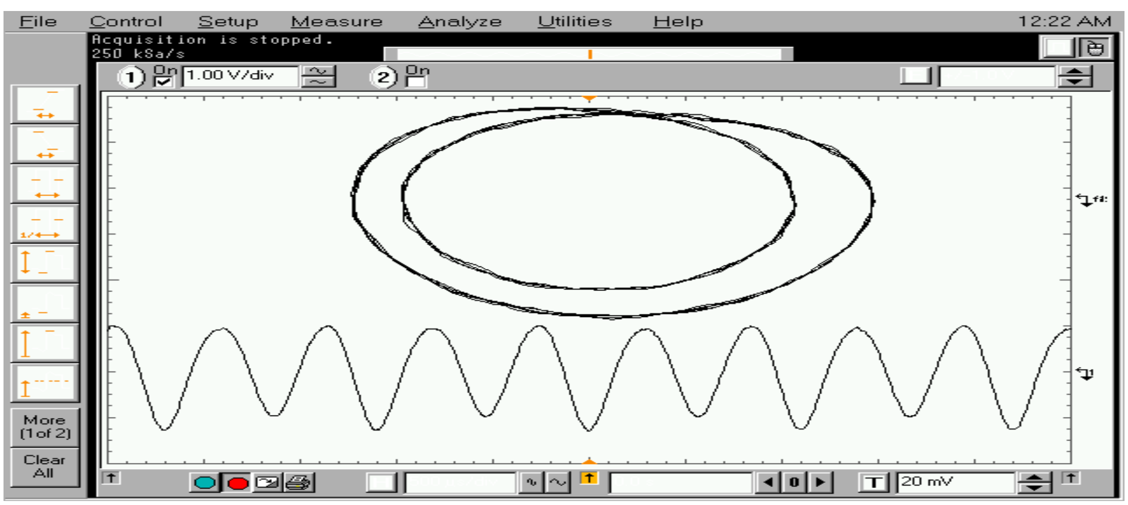

Figure 1 Period 2 , Phase space diagram and Real time behavior.

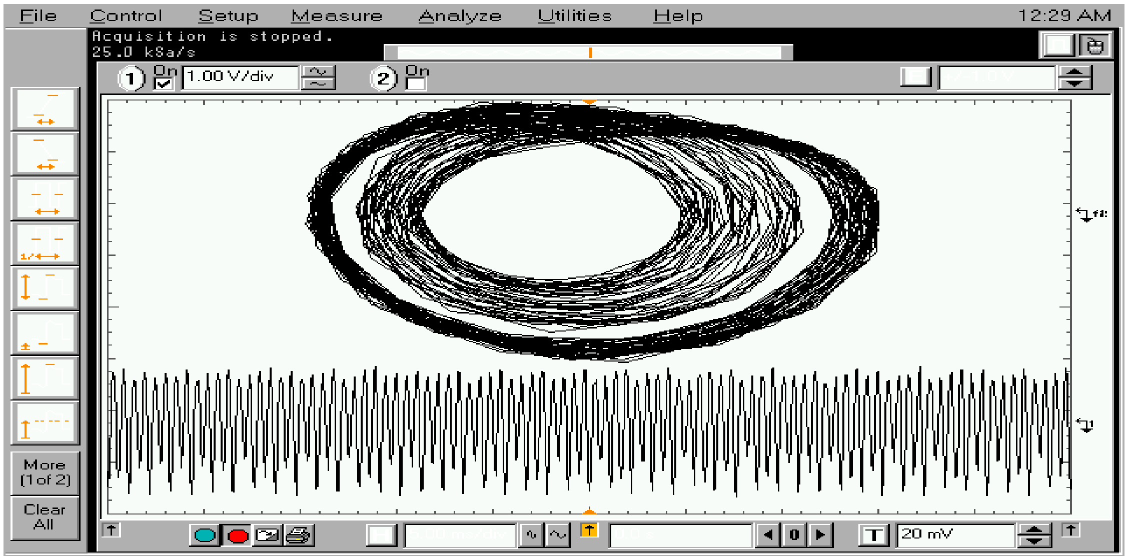

Figure 2. Chaotic behavior, Phase space and Real time, R=2.09

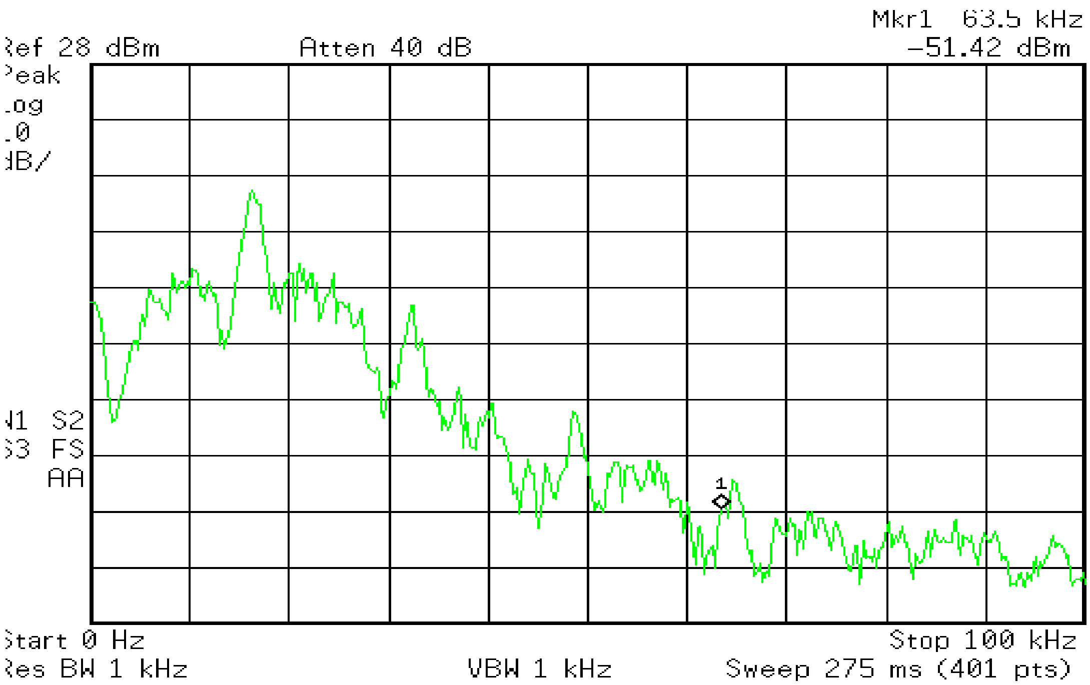

Figure 3. Power spectrum of the Chaotic X (*R*=2.09).

2. New chaotic circuit:

i) SAB-I

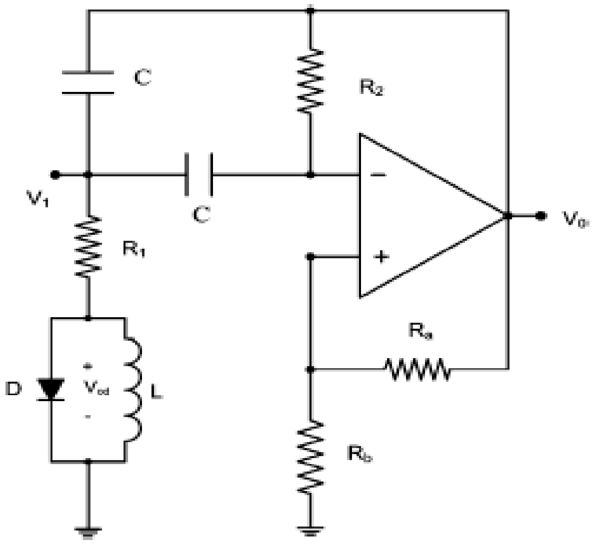

Fig4. Proposed circuit.

System equation:

$$
\begin{cases}
\frac{dx}{d\tau} = -2rx + (k+1)y - (k+1)p, \\
\frac{dy}{d\tau} = -r(\frac{2k+1}{k+1})x + ky - kp, \\
\frac{dz}{d\tau} = bp, \\
\varepsilon\frac{dp}{d\tau} = y - z - (1+a)p + a,
\end{cases}
$$

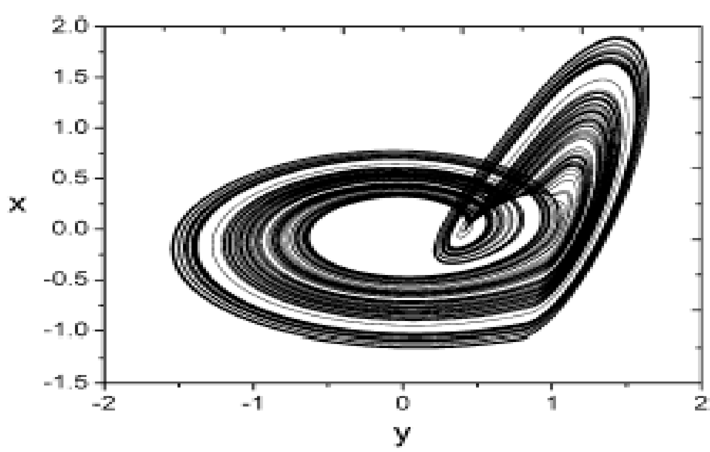

Fig5. Numerical result for the chaotic attractor.

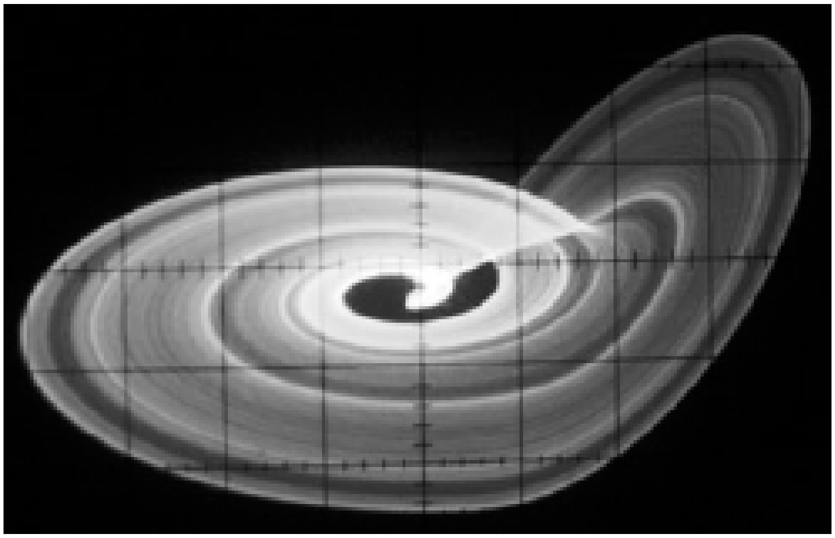

Figure 6. Chaotic behavior observed from experimental circuit.

SAB-II

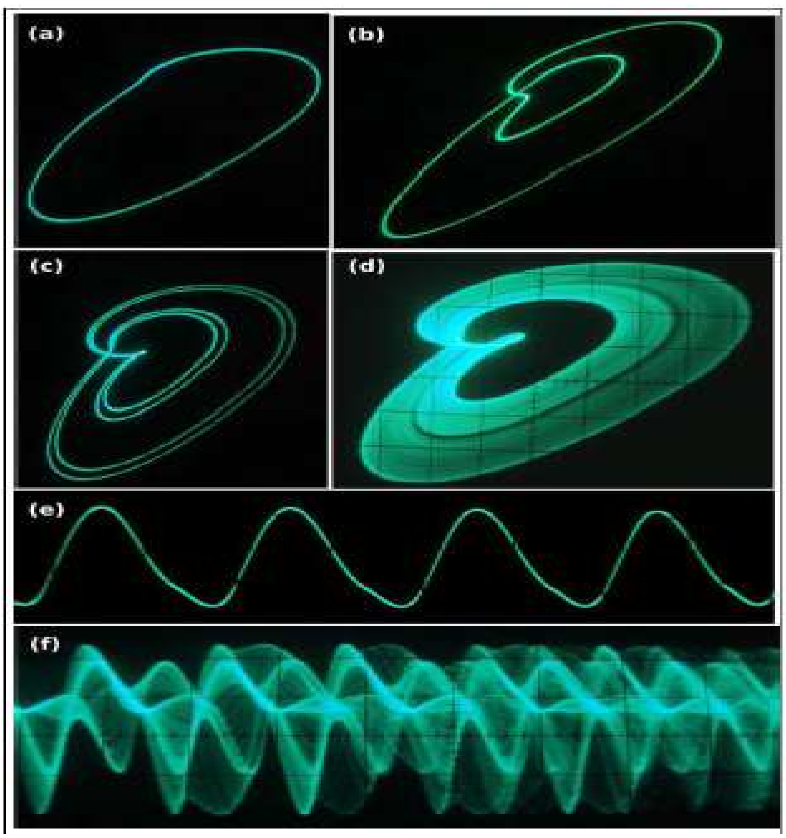

Fig7. Periodic and Chaotic signals from SAB-II circuit.

3. Lorenz circuit:

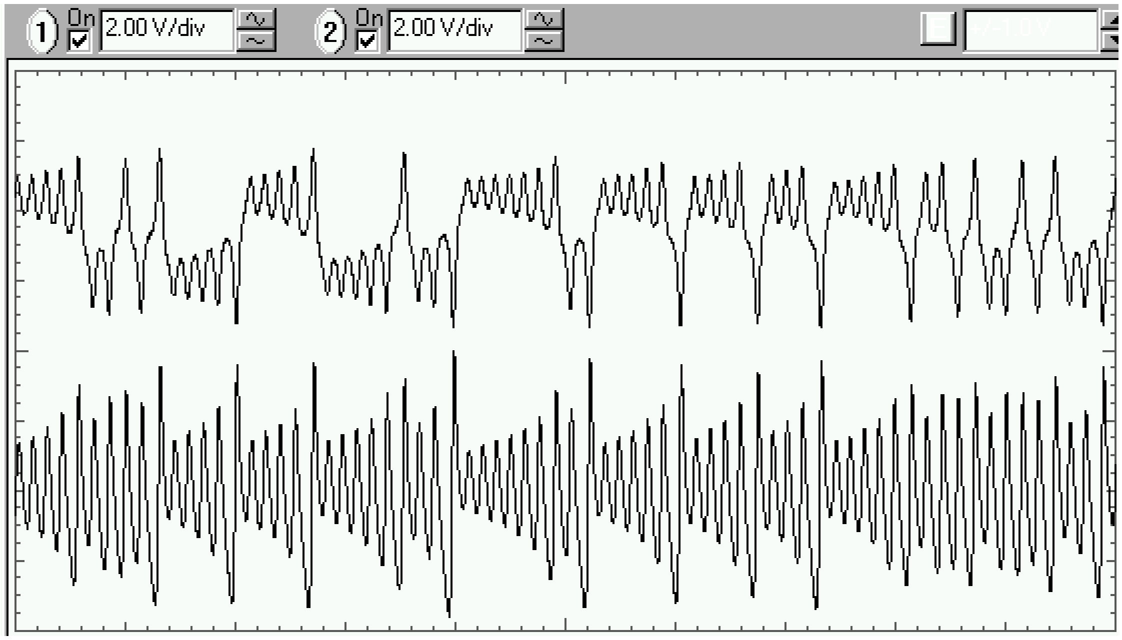

Fig. 8 . Chaotic behavior X (upper trace) and Z (lower trace). R4=22 kohm.

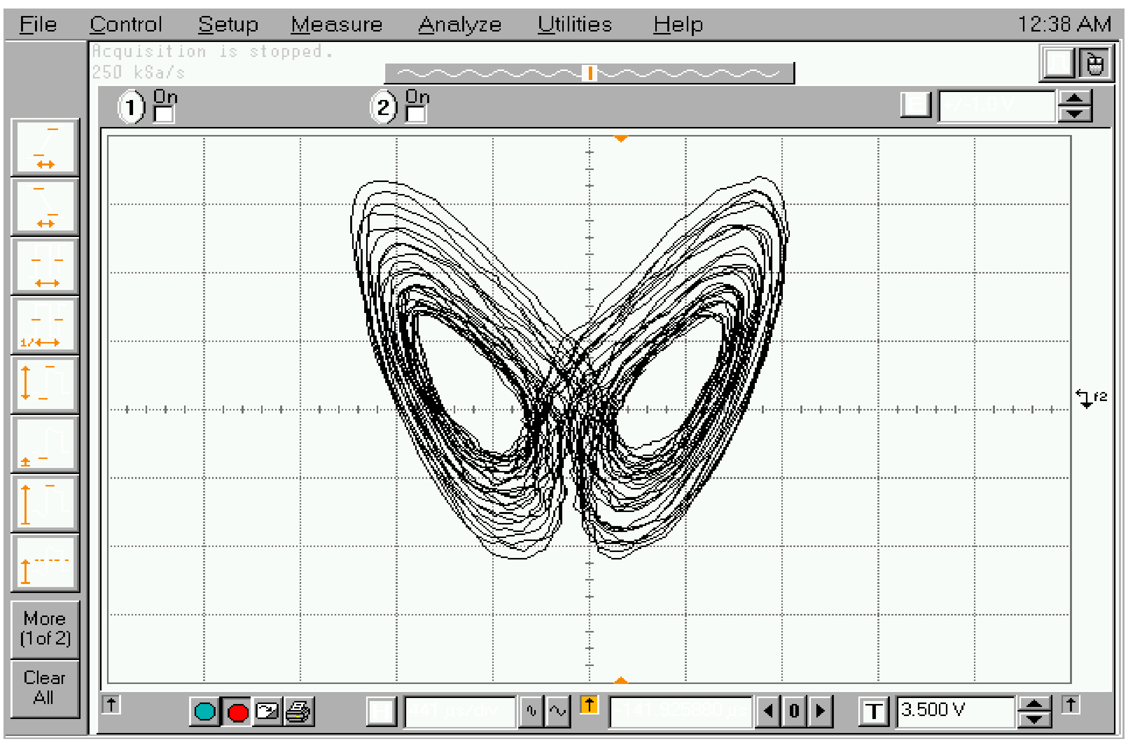

Fig. 9. Chaotic behavior shown in phase plane: X (horizontal) and Z (vertical).

4. Synchronization of chaos in Jerk circuit:

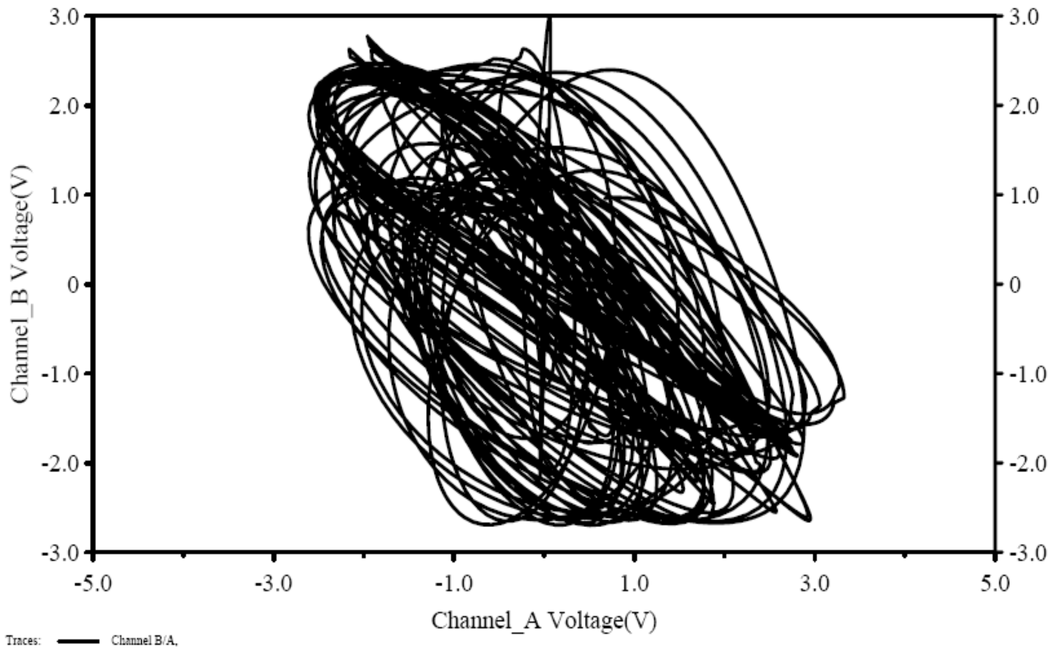

Fig. 10. Phase plot of xvs.u for R1=4Kohm.

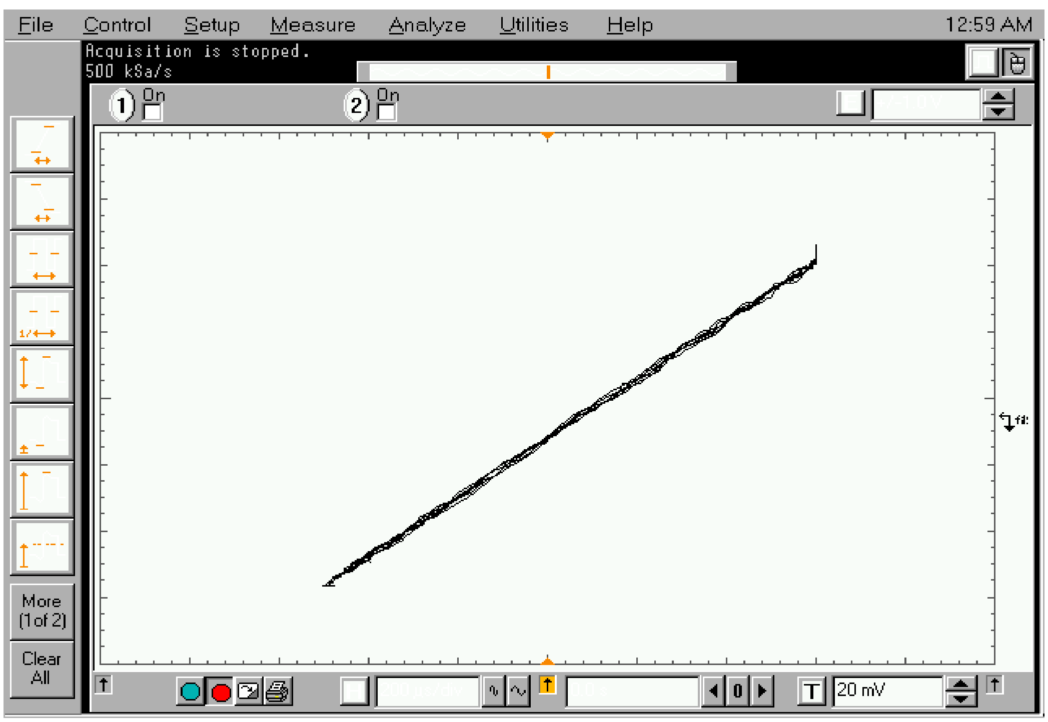

Fig. 11. Phase plot of xvs. u for R1=800 ohm.

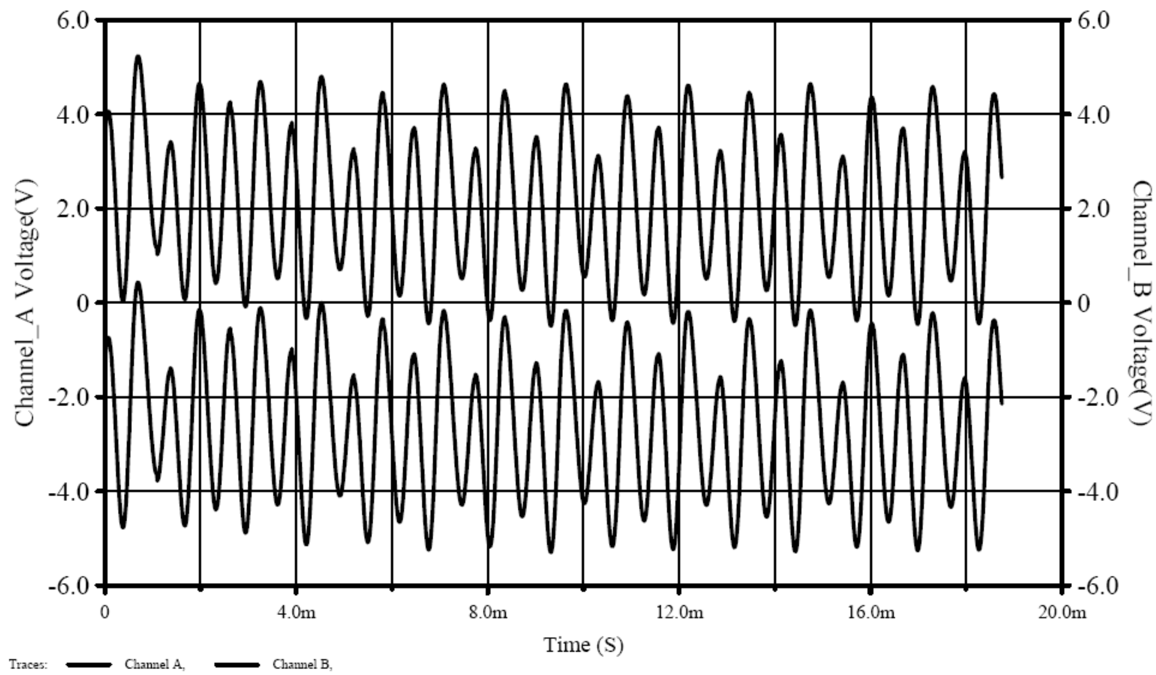

Fig.12. Real time plot of x and u for R1=800 ohm. Upper trace is for Response and lower trace is for Drive.

5. Chaotic communication system

The scheme is as follows: at first add the sinusoidal modulating signal with the chaotic y-variable of Drive circuit. Two Jerk circuits are made synchronized by using feedback synchronization as discussed earlier. Now (y+modulating) signal is subtracted by v-variable of the Response Jerk circuit. If two Jerk circuits are in a synchronized state then the subtracted output is simply the modulating signal. From the security point of view, if an unauthorized user does not know the x-variable then he can not reconstruct the modulating signal. Also since the modulating signal is masked by the chaotic signal it is not possible by any user to get an idea of the modulating signal.

Figure 13 shows the (y+sin(w.t)) wave form. Clearly one can not identify the sinusoidal signal masked in the chaotic signal.

Figure 14 shows the original sinusoidal signal (upper trace) and reconstructed modulating signal (lower trace).

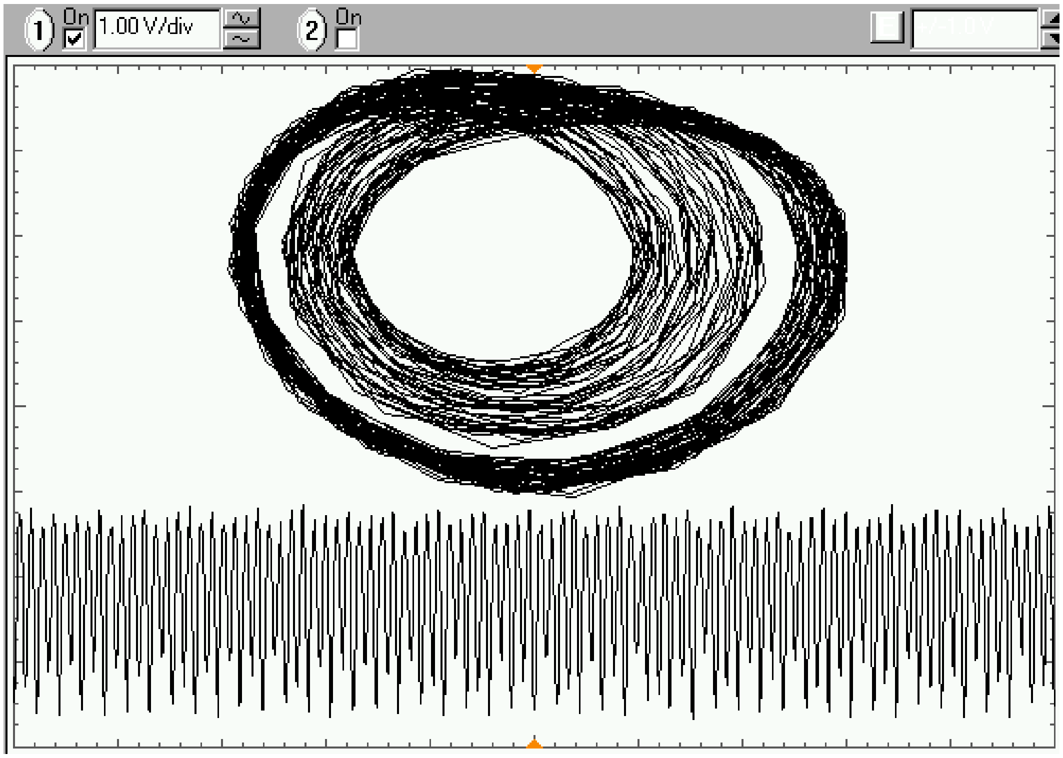

Figure.13. chaotic y variable plus sin(2.pi.f.t) (f=1kHz).

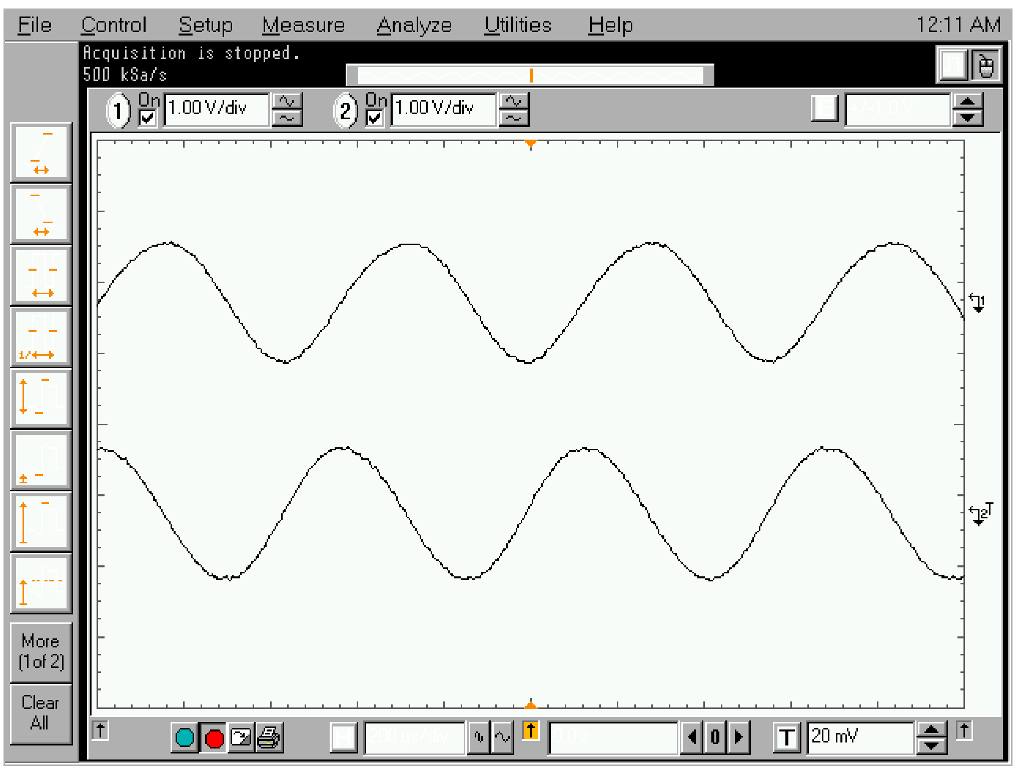

Figure. 14. Upper trace: sinusoidal modulating signal. Lower trace: reconstructed signal.

List of publication

(In all the papers the PI has acknowledged the University Grants Commission (UGC) India Project no: F No: 34-506/2008 (SR))

List of publication

Journal:

1. Tanmoy Banerjee, B. Karmakar and B C Sarkar. Single Amplifier Biquad Based Autonomous Electronic Oscillators for Chaos Generation. Vol.62, pp. 859-866, 2010, **Nonlinear Dynamics (Springer).**
2. Tanmoy Banerjee, B. Karmakar and B C Sarkar. Chaotic Electronic Oscillator from single amplifier biquad. Submitted to **AEU: International Journal of Electronics and Communications (Elsevier).**

Conferences:

1. Tanmoy Banerjee, B. Karmakar and B C Sarkar. A new autonomous simple electronic circuit for chaos generation. International Symposium on Complex Dynamics and Applications (CDSA-2009), Digha Science Center, Digha, 4-6 Dec, 2009.

2. Tanmoy Banerjee, B. Karmakar and B C Sarkar. Exploring the chaotic dynamics of an autonomous electronic circuit. Oral presentation in International Conference on Recent advances in Mathematics and its Applications (ICRAMA-2010), Department of Mathematics, Burdwan University, Burdwan, 13-15 January 2010.

3. T. Banerjee and J. Chatterjee. Chaotic communication exploiting feedback Synchronization in chaotic jerk circuit. International Conference on Radiation Physics and its Applications (ICRPA 2010), 16-17 January, 2010.

4. Tanmoy Banerjee, B. Karmakar and B C Sarkar. Design of a new autonomous chaotic electronic circuit. International Conference on Radiation Physics and its Applications (ICRPA 2010), 16-17 January, 2010.

Nonlinear Dyn (2010) 62: 859–866
DOI 10.1007/s11071-010-9768-5

ORIGINAL PAPER

Single amplifier biquad based autonomous electronic oscillators for chaos generation

Tanmoy Banerjee · B. Karmakar · B.C. Sarkar

Received: 14 December 2009 / Accepted: 22 June 2010 / Published online: 17 July 2010

Abstract Two simple autonomous chaotic electronic circuits have been proposed in this paper. The core of each of the circuits consists of a single amplifier biquad (SAB). We have proposed two configurations of converting this SAB into chaotic oscillators using suitable passive nonlinear element and a storage element in the form of an inductor. The mathematical models of the proposed chaotic circuits have been constructed, which are fourth order autonomous nonlinear differential equations. The behavior of the proposed circuits has been investigated through numerical simulations, Spice-based circuit simulations and electronic hardware experiments and they agree well with each other. It has been found that both the circuits show complex behaviors like bifurcations and chaos for a certain range of circuit parameters.

Keywords Single amplifier biquad · Chaotic electronic circuits · Chaos · Bifurcations

1 Introduction

Studies on nonlinear dynamical problems of different physical systems have been attracting the attention of researchers for at least three decades [1]. An electronic Chua circuit first showed that chaos is not a mathematical abstraction but can be observed in a laboratory [2]. Since then, research in chaotic electronic circuits and systems has been boosted up owing to its application potentiality in secure electronic communication systems [3].

Chaotic Colpitts oscillator proposed by Kennedy [4] showed that a sinusoidal oscillator can be brought into chaotic regime. Namajunas and Tamaševičius [5] modified a Wien-bridge circuit for chaos generation. Later Elwakil and Kennedy reported [6] a semisystematic procedure of modifying a sinusoidal oscillator into a chaotic oscillator by introducing a storage element (inductor or capacitor) and a suitable passive nonlinearity (in the form of a diode or field effect transistor). Many chaotic oscillators based on these modifications have been reported (e.g., Wien-bridge oscillator [6, and references therein], Twin-T oscillator [7], current feedback op amp based oscillators [8], etc.). Also, efforts are there to find out high frequency chaotic oscillations from sinusoidal oscillators [9]. At the same time, investigation on chaotic attractors from electronic circuits incorporating different kind of nonlinearity has been reported [10].

In this paper, we have proposed two new autonomous chaotic electronic oscillators. We have designed a single amplifier sinusoidal oscillator (SASO) based on a single amplifier biquad (SAB) proposed by Deliyannis and Friend [11–13] and modified it for generating chaotic oscillations using a diode as a passive nonlinear element (unlike chaotic Chua oscilla-

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tor, where a locally active nonlinear element is necessary) and a storage element in the form of an inductor. The proposed chaotic circuits have been mathematically modeled, which are a set of four first order coupled nonlinear differential equations involving voltages and currents with piecewise linear nonlinearity. The behavior of the proposed circuits have been investigated through numerical simulations using the fourth order Runge–Kutta method, Spice-based circuit simulator, and electronic hardware experiments and they agree well with each other. Through the phase plane plots, bifurcation diagrams and experimentally obtained power spectra it has been ensured that the circuits show complex behaviors like bifurcations and chaos for certain range of parameter values.

2 SAB based sinusoidal oscillator

SAB based high-Q active band pass filter (BPF) [11, 12] has been shown in Fig. 1. It consists of an op amp, two capacitors having same value C and four resistors. It is a second order system and its transfer function is given by

$$H(s) = \frac{-(k+1)/(R_1 C)s}{s^2 + (2/R_2 C - k/R_1 C)s + 1/R_1 R_2 C^2}, \quad (1)$$

where $k = R_b/R_a$. This SAB can be converted into a single amplifier sinusoidal oscillator (SASO) by con-

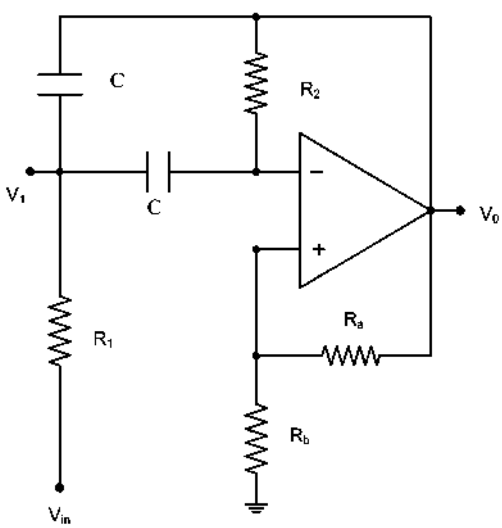

Fig. 1 Circuit diagram of a SAB. It consists of an op amp, resistors R_1, R_2, R_a, R_b and two capacitors having capacitance C. This can be converted into a SASO by connecting V_{in} terminal to the ground

necting the terminal V_{in} (as shown in Fig. 1) to the ground [13]. Proper choice of R_b (and hence k) makes the coefficient of s in the denominator negative, which in turn brings the pole of the circuit to the right half of the s-plane. The SASO can be described by the following two sets of coupled first order autonomous differential equations in voltages V_0 and V_1 in terms of circuit parameters,

$$\begin{cases} C\frac{dV_0}{dt} = -\frac{2}{R_2}V_0 + \frac{(k+1)}{R_1}V_1, \\ C\frac{dV_1}{dt} = -\frac{(2k+1)}{(k+1)R_2}V_0 + \frac{k}{R_1}V_1. \end{cases} \quad (2)$$

Equation (2) can be written as

$$\begin{pmatrix} \frac{dV_0}{dt} \\ \frac{dV_1}{dt} \end{pmatrix} = \begin{pmatrix} -2/CR_2 & (k+1)/CR_1 \\ -(2k+1)/(k+1)CR_2 & k/CR_1 \end{pmatrix} \times \begin{pmatrix} V_0 \\ V_1 \end{pmatrix} = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix} \begin{pmatrix} V_0 \\ V_1 \end{pmatrix}. \quad (3)$$

Now for any sinusoidal oscillator given in (3) one can find the condition of oscillation by making $\alpha_{11} + \alpha_{22} = 0$, which gives the condition of oscillation of the SASO as

$$k = \frac{2R_1}{R_2}. \quad (4)$$

Further the frequency of oscillation can be derived as, $\omega_0 = \sqrt{(\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21})}$, which gives the frequency of oscillation of the SASO as

$$\omega_0 = \frac{1}{C\sqrt{R_1 R_2}}. \quad (5)$$

3 Modification of SASO for chaos generation

In this section, we describe two modifications of the SASO for generating chaos.

3.1 First modification

Figure 2 shows the circuit diagram of the proposed modified chaotic SASO (MSASO-1). A parallel Diode–Inductor (DL) arrangement [6] has been introduced between the V_1 terminal (through a resistance R_1) and ground. The diode switches on and

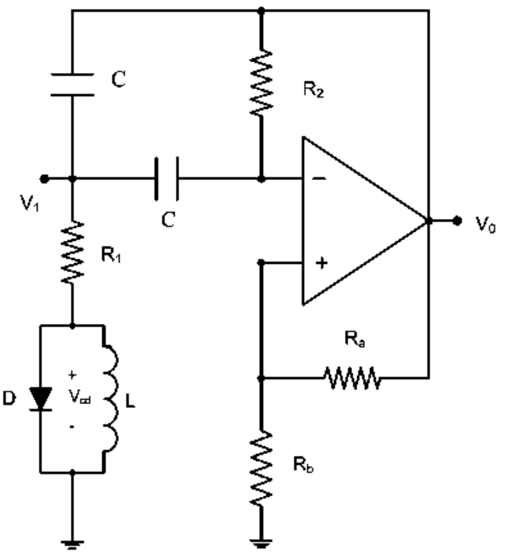

Fig. 2 Circuit diagram of first modification (MSASO-1). A parallel Diode (D)–Inductor (L) arrangement has bean introduced between V_1 (through resistor R_1) and ground

off according the voltage developed across the inductor [6]. This inductor voltage appears across diode parasitic transit capacitance C_D and let that voltage be V_{CD}. The circuit behavior can be modeled by a set of four first order coupled autonomous differential equations in voltages (V_0, V_1 and V_{CD}) and inductor current I_L

$$\begin{cases} C\frac{dV_0}{dt} = -\frac{2}{R_2}V_0 + \frac{(k+1)}{R_1}V_1 - \frac{(k+1)}{R_1}V_{\mathrm{CD}}, \\ C\frac{dV_1}{dt} = -\frac{(2k+1)}{(k+1)R_2}V_0 + \frac{k}{R_1}V_1 - \frac{k}{R_1}V_{\mathrm{CD}}, \\ \frac{dI_L}{dt} = \frac{V_{\mathrm{CD}}}{L}, \\ C_D\frac{dV_{\mathrm{CD}}}{dt} = \frac{1}{R_1}V_1 - I_L - \frac{1}{R_1}V_{\mathrm{CD}} - I_D. \end{cases} \tag{6}$$

The nonlinear diode current I_D has been modeled as a piece-wise linear function of voltage such that

$$\begin{aligned} I_D &= \frac{1}{R_D}(V_{\mathrm{CD}} - V_\gamma), \quad \text{if } V_{\mathrm{CD}} \geq V_\gamma, \\ &= 0, \quad \text{if } V_{\mathrm{CD}} < V_\gamma. \end{aligned} \tag{7}$$

R_D is the diode forward conductance resistance and V_γ is the diode forward voltage drop.

The sets of equations given in (6) and (7) can be written in the following dimensionless form:

$$\begin{cases} \frac{dx}{d\tau} = -2rx + (k+1)y - (k+1)p, \\ \frac{dy}{d\tau} = -r(\frac{2k+1}{k+1})x + ky - kp, \\ \frac{dz}{d\tau} = bp, \\ \varepsilon\frac{dp}{d\tau} = y - z - (1+a)p + a, \end{cases} \tag{8a}$$

where we have defined the following dimensionless quantities: $\tau = t/R_1C$, $x = V_0/V_\gamma$, $y = V_1/V_\gamma$, $p = V_{\mathrm{CD}}/V_\gamma$, $z = R_1I_L/V_\gamma$, $r = R_1/R_2$, $b = R_1^2C/L$, $\varepsilon = C_D/C$, $K_D = R_1/R_D$. The quantity a is defined as

$$\begin{aligned} a &= K_D, \quad \text{if } p \geq 1, \\ &= 0, \quad \text{if } p < 1. \end{aligned} \tag{8b}$$

Equilibrium point of (8) is given by $(x_0, y_0, z_0, p_0) = (0, 0, a, 0)$. Therefore, in the region $p < 1$ (i.e., for $a = 0$), there is a single equilibrium point. In the region $p \geq 1$, the equilibrium point is virtual, meaning that it lies outside this region [6].

Although the system is described by a fourth-order differential equation, it effectively lives in a three-dimensional subspace owing to the small value of C_D (i.e., small value of ε) [6]. To explore the dynamics of the system, numerical integration has been carried out on the sets of (8) using fourth-order Runge–Kutta algorithm with step size $h = 0.001$. Figure 3 shows the phase plane representation (in y–x plane) of the system for different values of the control parameter r keeping the values of other parameters fixed at $k = 0.48$, $b = 0.01$, $\varepsilon = 0.1$, $K_D = 20$. For $r > 0.49$, it shows a single trivial equilibrium point at $(0, 0, 0, 0)$; at $r = 0.49$ it goes to a limit cycle through Hopf bifurcation. Figure 3(a) shows a limit cycle at $r = 0.45$. A decrease in the value of r makes the period one behavior to a distorted period one oscillation [14]. Further decrease of value of r results in a period doubling route to chaos. The chaotic attractor at $r = 0.16$ has been shown in Fig. 4.

Figure 5 shows the bifurcation diagram in x by plotting the values of x (excluding the transients) from the Poincare section at $y = -0.2$, which agrees well with the phase plane plots. Studies on the detailed dynamical behavior of this system are in progress and will be reported elsewhere.

The circuit has been studied using Spice-based circuit simulator. Op amp used in this simulation is TL082 with ±12 volt power supply and general-purpose diode (1N1183) has been chosen. Figure 6 shows the chaotic behavior in the V_1–V_0 space for: $R_1 = 100\ \Omega$, $R_2 = 3.1\ \mathrm{k}\Omega$, $R_a = 2\ \mathrm{k}\Omega$, $R_b = 500\ \Omega$, $C = 1$ nF, and $L = 7.8$ mH. It can be seen from Figs. 4 and 6 that numerical results and Spice-based circuit simulation results agree well with each other. Experimental results will be discussed in Sect. 4.

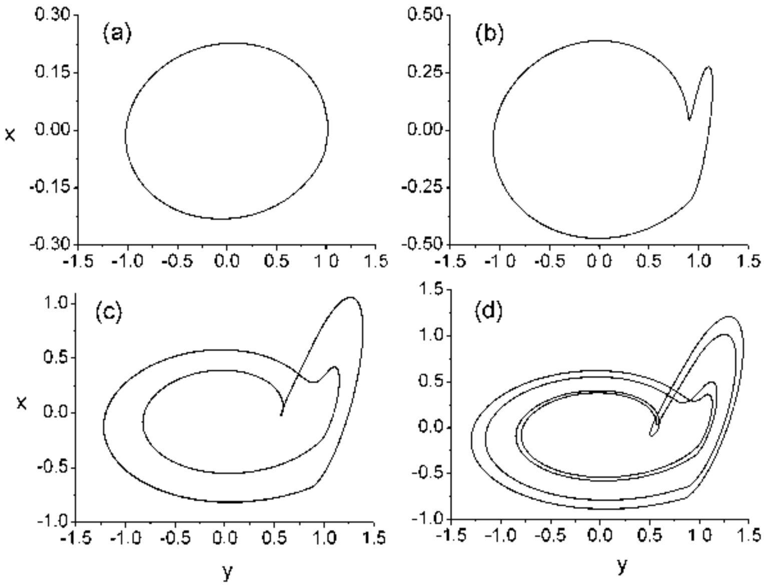

Fig. 3 Phase space representation of the system dynamics for (**a**) $r = 0.45$ (period-1), (**b**) $r = 0.25$ (distorted period-1), (**c**) $r = 0.18$ (period-2), (**d**) $r = 0.175$ (period-4). The values of other parameters are $k = 0.48$, $b = 0.01$, $\varepsilon = 0.1$, $K_D = 20$

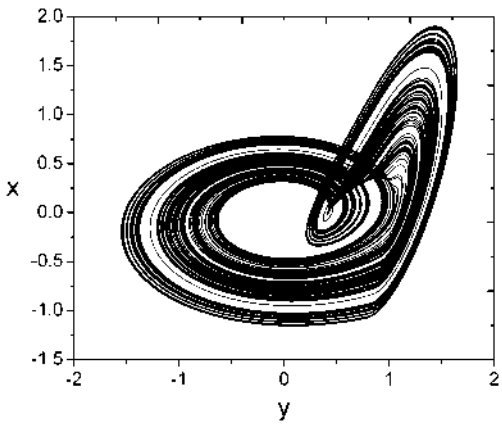

Fig. 4 Chaotic attractor in y–x space at $r = 0.16$ ($k = 0.48$, $b = 0.01$, $\varepsilon = 0.1$ and $K_D = 20$)

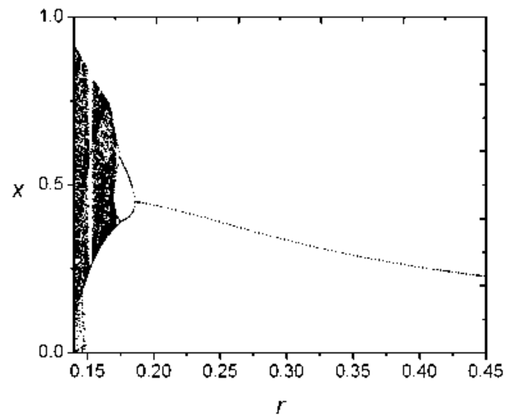

Fig. 5 Bifurcation diagram of MSASO-1 taking r as the control parameter ($k = 0.48$, $b = 0.01$, $\varepsilon = 0.1$ and $K_D = 20$)

3.2 Second modification

Figure 7 shows the second modification of SASO (MSASO-2) for generating chaos. In this modification, the Diode-Inductor arrangement is introduced between the noninverting terminal of the op amp (through a resistance R_b) and the ground. The circuit behavior can be modeled by a set of four first order coupled autonomous differential equations in voltages V_0, V_1, and V_{CD} and inductor current I_L, which are as follows:

$$\begin{cases} C\frac{dV_0}{dt} = (\frac{\beta}{R_b} - \frac{2}{R_2})V_0 + \frac{(k+1)}{R_1}V_1 - \varepsilon' I_L \\ \qquad + (\frac{2}{R_2} - \frac{\beta}{R_b})V_{\mathrm{CD}} - \varepsilon' I_D, \\ C\frac{dV_1}{dt} = (\frac{\beta}{R_b} - \frac{(2k+1)}{(k+1)R_2})V_0 + \frac{k}{R_1}V_1 - \varepsilon' I_L \\ \qquad + (\frac{(2k+1)}{R_2(k+1)} - \frac{\beta}{R_b})V_{\mathrm{CD}} - \varepsilon' I_D, \\ \frac{dI_L}{dt} = \frac{V_{\mathrm{CD}}}{L}, \\ C_D\frac{dV_{\mathrm{CD}}}{dt} = \frac{k}{R_b(k+1)}V_0 - I_L - \frac{k}{R_b(k+1)}V_{\mathrm{CD}} - I_D, \end{cases} \tag{9}$$

where $k = R_b/R_a$, $\varepsilon' = C/C_D$, $\beta = \varepsilon' k/(k+1)$. Further, the diode current I_D has been modeled as the same piece-wise linear form as given in (7).

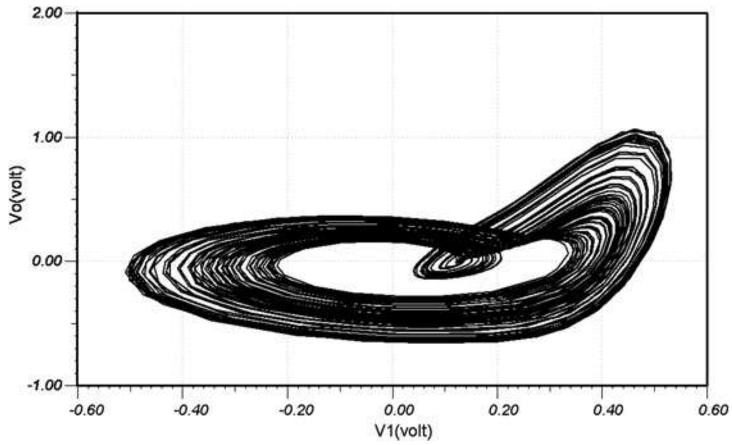

Fig. 6 Spice-based circuit simulation result: the chaotic behavior in the V_1–V_0 space for: $R_1 = 100\ \Omega$, $R_2 = 3.10\ \Omega$, $R_a = 2\ \mathrm{k}\Omega$, $R_b = 500\ \Omega$, $C = 1$ nF, and $L = 7.8$ mH

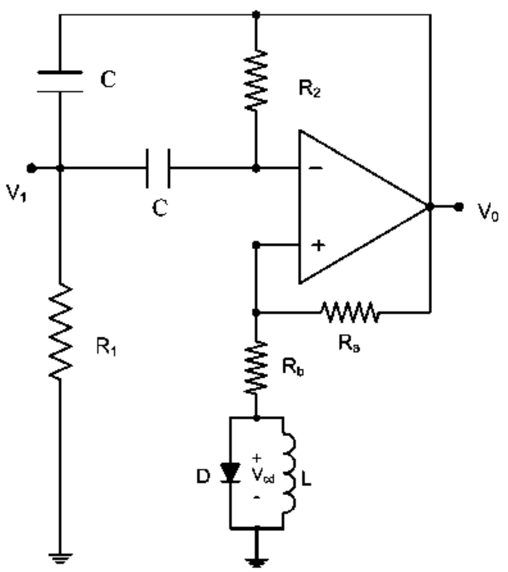

Fig. 7 Circuit diagram of second modification (MSASO-2). A parallel Diode (D)–Inductor (L) arrangement has bean introduced between the noninverting terminal (through resistor R_b) and ground

By introducing the following dimensionless quantities: $\tau = t/R_bC$, $x = V_0/V_\gamma$, $y = V_1/V_\gamma$, $p = V_{\mathrm{CD}}/V_\gamma$, $z = R_bI_L/V_\gamma$, $r_1 = R_b/R_1$, $r_2 = R_b/R_2$, $b = R_b^2C/L$, $\varepsilon = C_D/C$, $K_D = R_b/R_D$, one can write (9) in the following dimensionless form:

$$\begin{cases} \frac{dx}{d\tau} = (\beta - 2r_2)x + r_1(k+1)y - \varepsilon' z \\ \qquad + (2r_2 - \beta - \varepsilon' a)p + \varepsilon' a, \\ \frac{dy}{d\tau} = (\beta - (\frac{2k+1}{k+1})r_2)x + r_1 ky - \varepsilon' z \\ \qquad + ((\frac{2k+1}{k+1})r_2 - \beta - \varepsilon' a)p + \varepsilon' a, \\ \frac{dz}{d\tau} = bp, \\ \varepsilon\frac{dp}{d\tau} = \frac{k}{(k+1)}x - z - (a + \frac{k}{k+1})p + a. \end{cases} \tag{10}$$

The quantity a is defined by (8b).

In this case also, the equilibrium point is $(x_0, y_0, z_0, p_0) = (0, 0, a, 0)$. Therefore, in the region $p < 1$ (i.e., for $a = 0$), there is a single equilibrium point. In the region $p \geq 1$, the equilibrium point is virtual, meaning that it lies outside this region [6].

As discussed in the earlier section, the system is effectively living in a three-dimensional subspace owing to the small value of C_D (i.e., small value of ε). The sets of (10) along with (8b) have been numerically integrated using fourth order Runge–Kutta algorithm with step size $h = 0.001$. Figure 8 shows the phase space trajectory for different values of r_1 (with $r_2 = 0.16$, $k = 0.02$, $K_D = 10$, $b = 0.01$, $\varepsilon = 0.1$, and $\varepsilon' = 10$) which shows a period doubling route to chaos with increasing r_1. Figure 9 shows the bifurcation diagram of the system variable x (plotted using Poincare map at $y = 1$), which agrees well with the phase plane plots.

The circuit of MSASO-2 has also been tested in Spice-based electronic circuit simulator using TL082 op amp with ±12 volt power supply and 1N1183 diode. Figure 10 shows the chaotic behavior for: $R_a = 2\ \mathrm{k}\Omega$, $R_b = 440\ \Omega$, $R_1 = 200\ \Omega$, $R_2 = 700\ \Omega$, $C = 1$ nF, and $L = 7.8$ mH.

4 Experimental results

Electronic hardware experiments have been carried out using discrete components for both the MSASO-1 and MSASO-2 circuits. Op amps used in the experiments are TL082 with ±12 volt power supply; diodes are general purpose 1N1183 diodes. Following values of components are used throughout the experiments:

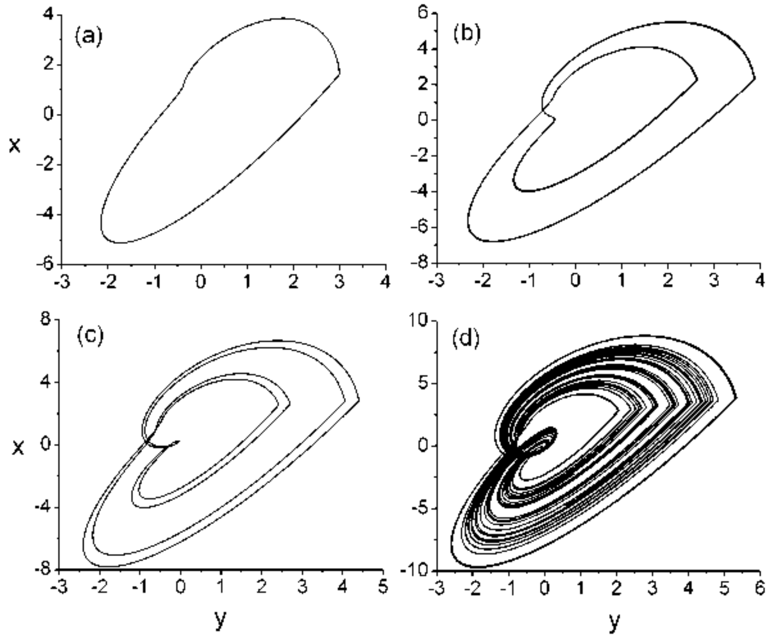

Fig. 8 Phase space trajectory for different r_1 (**a**) $r_1 = 0.5$ (period-1), (**b**) $r_1 = 0.65$ (period-2), (**c**) $r_1 = 0.74$ (period-4), (**d**) $r_1 = 0.88$ (chaos) (with $r_2 = 0.16$, $k = 0.02$, $K_D = 10$, $b = 0.01$, $\varepsilon = 0.1$, and $\varepsilon' = 10$)

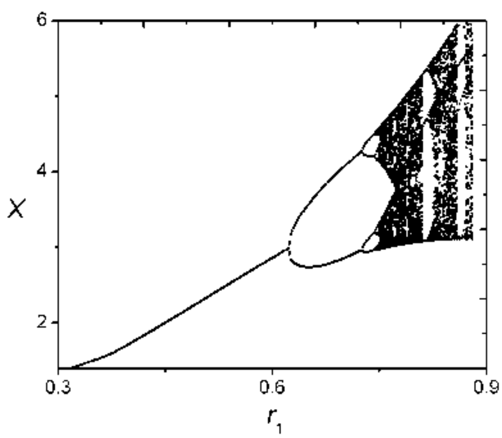

Fig. 9 Bifurcation diagram of MSASO-2 taking r_1 as the control parameter (with $r_2 = 0.16$, $k = 0.02$, $K_D = 10$, $b = 0.01$, $\varepsilon = 0.1$, and $\varepsilon' = 10$)

$C = 1$ nF ($\pm 5\%$) and $L = 7.8$ mH ($\pm 10\%$). All the resistors used in the experiments have $\pm 5\%$ tolerance.

For the MSASO-1 circuit, we have fixed the values of resistances $R_a = 2$ kΩ, $R_b = 500$ Ω, and $R_1 = 100$ Ω (using a 1 kΩ potentiometer (POT)). R_2 has been varied (using a 5 kΩ POT) for exploring different behavior of the circuit. With the increasing resistance R_2, the circuit shows a transition from a dc state to an oscillatory behavior (a period-1 oscillations) of frequency 34.56 kHz. An increase in R_2 beyond 1 kΩ results in a distorted sinusoidal oscillation. A period doubling route to chaos has been observed for further increase in R_2. Figure 11 shows the oscilloscope trace, which is a chaotic attractor in the V_1–V_0 space for $R_2 = 3.10$ kΩ. Figure 12 shows the experimental power spectrum (measured with Agilent E4411B spectrum analyzer) of the chaotic signal of Fig. 11 and it is broad in nature, which is one of the characteristics of a chaotic oscillation.

For MSASO-2, we take $R_a = 2$ kΩ, $R_1 = 200$ Ω (POT), and $R_2 = 700$ Ω (POT). By increasing R_b through a 2 kΩ POT a period doubling route to chaos has been observed (with a period-1 oscillation of frequency 114.7 kHz). Figure 13 shows the oscilloscope trace of the chaotic attractor in the V_1–V_0 space for $R_b = 440$ Ω. Figure 14 shows the corresponding experimental power spectrum of the chaotic signal, which is significantly broad in nature.

5 Conclusion

In this paper, we have introduced two new autonomous chaotic electronic oscillators. The circuits are simple as they require only a single amplifier as an active element and one passive nonlinearity in the form of a general-purpose diode. These circuits have been mathematically described by a set of four first order coupled nonlinear autonomous differential equations. Numerical integrations show that the oscillators can be driven

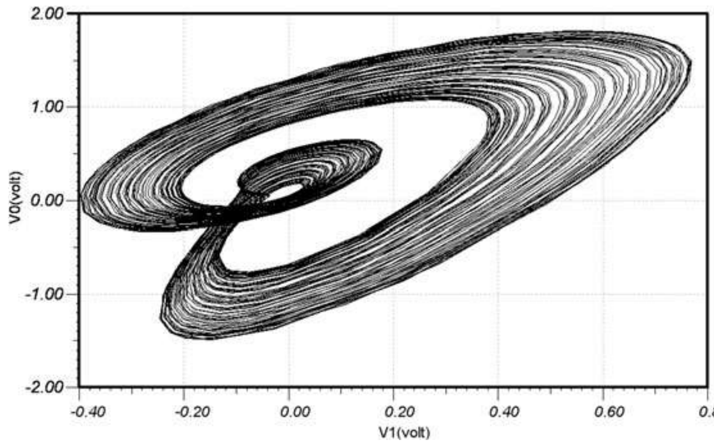

Fig. 10 Spice-based electronic circuit simulation result. It shows that the circuit shows chaotic behavior for: $R_a = 2\ \text{k}\Omega$, $R_1 = 200\ \Omega$ and $R_2 = 700\ \Omega$ and $R_b = 440\ \Omega$, $C = 1$ nF and $L = 7.8$ mH

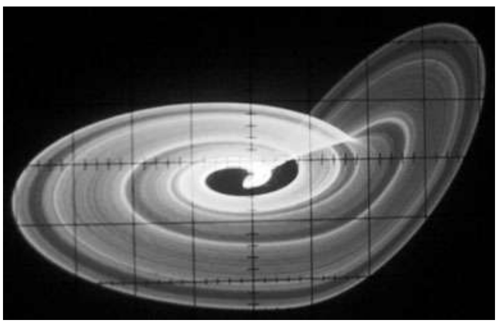

Fig. 11 The oscilloscope trace of the chaotic attractor in MSASO-1 in the V_1–V_0 space at $R_1 = 400\ \Omega$ (POT), $R_2 = 3.3\ \text{k}\Omega$ (POT), $R_a = 1\ \text{k}\Omega$, $R_b = 476\ \Omega$, $C = 1$ nF, and $L = 7.8$ mH. V_1 (x-axis): 0.2 v/div, V_0 (y-axis): 0.5 v/div

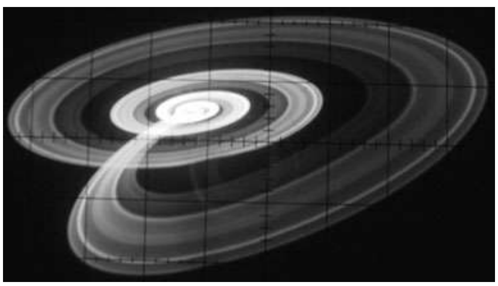

Fig. 13 The oscilloscope trace of the chaotic attractor in MSASO-2 in the V_1–V_0 space at $R_a = 2\ \text{k}\Omega$ (POT), $R_b = 440\ \Omega$ (POT) and $R_1 = 200\ \Omega$ (POT) and $R_2 = 700\ \Omega$ (POT), $C = 1$ nF, and $L = 7.8$ mH. V_1 (x-axis): 0.2 v/div, V_0 (y-axis): 0.5 v/div

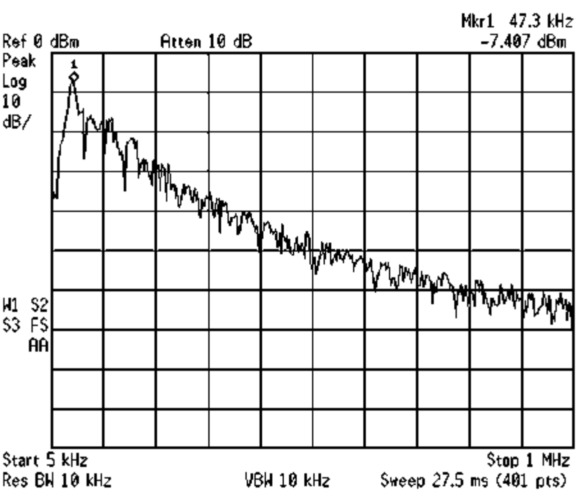

Fig. 12 Experimental power spectrum of chaotic oscillation from V_1 of MSASO-2 (parameters are same as Fig. 11). (Frequency span: 5 kHz to 1 MHz)

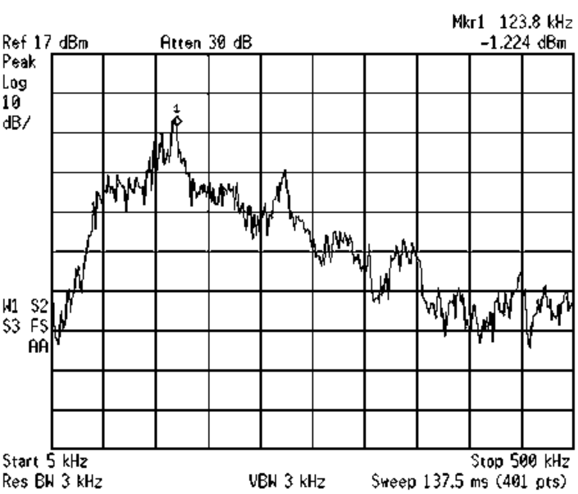

Fig. 14 Spectrum of chaotic oscillation from V_1 of MSASO-2 (parameters are same as Fig. 13). (Frequency span: 5 kHz to 500 kHz)

into chaotic region for some suitable circuit parameters. Simulations in the Spice-based circuit simulator and real world hardware experiments agree well with the numerical results. However, realization of high frequency chaotic signals [9, 15] from these circuits is limited by the bandwidth limitations of the op amp used.

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