SYLLABUS

For

M. A./M.Sc. in MATHEMATICS

Four Semester

(Effective from the academic session 2007 – 2008 and onwards)

THE UNIVERSITY OF BURDWAN
RAJBATI, BURDWAN
WEST BENGAL
Duration of P.G. course of studies in Mathematics shall be two years with Semester-I, Semester-II, Semester-III and Semester-IV each of six months duration leading to Semester-I, Semester-II, Semester-III and Semester-IV examination in Mathematics at the end of each semester. Syllabus for P.G. courses in Mathematics is hereby framed according to the following schemes and structures.

**Scheme:** Total Marks = 1000 with 250 Marks in each semester comprising of five papers in each semester with 50 marks in each paper. In each theoretical paper 10% Marks is allotted for Internal Assessment. There are four theoretical papers and one practical paper in Semester-II. All students admitted to P.G. course in Mathematics shall take courses of Semester-I and Semester-II; and from the Semester-III they will opt for either Pure Stream or Applied Stream in Mathematics. The option norm is to be framed by the Department in each year. In each stream of Semester-III, the first three papers are general and the last two papers are special papers. In each stream of Semester-IV, the first two papers are general and the next two papers are special papers and the fifth paper is the term paper. In the Semester-III and Semester-IV, the Department will offer a cluster of special papers and the students will have to choose one according to the norms to be decided by the Department. The term paper is related with the respective special papers and the mark distribution is 30 for written submission, 15 for Seminar presentation and 5 for Viva-Voice. It should be noted that some special papers will be included in future as per discretion of the Department.
### COURSE STRUCTURE

**SEMESTER STRUCTURE OF M.A./M.SC. SYLLABUS IN MATHEMATICS**  
(SEMESTER-I, II, III, IV)

#### SEMESTER-I

- **Duration:** 6 Months  
- **Total Marks:** 250  
- **Total No. of Lectures:** 50 Hours per paper

Each Theoretical Paper Containing 45 Marks and 5 Marks for Internal Assessment

<table>
<thead>
<tr>
<th>Paper</th>
<th>Marks</th>
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<tbody>
<tr>
<td>MCG101:</td>
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</tr>
<tr>
<td>Functional Analysis-I (27+3)</td>
<td>30</td>
</tr>
<tr>
<td>Real Analysis-I (18+2)</td>
<td>20</td>
</tr>
<tr>
<td>MCG102:</td>
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<tr>
<td>Linear Algebra-I (27+3)</td>
<td>30</td>
</tr>
<tr>
<td>Modern Algebra-I (18+2)</td>
<td>20</td>
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<tr>
<td>MCG103:</td>
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<tr>
<td>Elements of General Topology (27+3)</td>
<td>30</td>
</tr>
<tr>
<td>Complex Analysis-I (18+2)</td>
<td>20</td>
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<tr>
<td>MCG104:</td>
<td></td>
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<tr>
<td>Ordinary Differential Equations &amp; Special Functions (27+3)</td>
<td>30</td>
</tr>
<tr>
<td>Operations Research-I (18+2)</td>
<td>20</td>
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<tr>
<td>MCG105:</td>
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<tr>
<td>Principle of Mechanics-I (27+3)</td>
<td>30</td>
</tr>
<tr>
<td>Numerical Analysis (18+2)</td>
<td>20</td>
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</table>

#### SEMESTER-II

- **Duration:** 6 Months  
- **Total Marks:** 250  
- **Total No. of Lectures:** 50 Hours per paper

Each Theoretical Paper Containing 45 Marks and 5 Marks for Internal Assessment

<table>
<thead>
<tr>
<th>Paper</th>
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<tbody>
<tr>
<td>MCG201:</td>
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<tr>
<td>Complex Analysis-II (27+3)</td>
<td>30</td>
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<tr>
<td>Real Analysis-II (18+2)</td>
<td>20</td>
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<tr>
<td>MCG202:</td>
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<tr>
<td>Partial Differential Equations (27+3)</td>
<td>30</td>
</tr>
<tr>
<td>Differential Geometry (18+2)</td>
<td>20</td>
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<tr>
<td>MCG203:</td>
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<tr>
<td>Operations Research-II (27+3)</td>
<td>30</td>
</tr>
<tr>
<td>Principle of Mechanics-II (18+2)</td>
<td>20</td>
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<tr>
<td>MCG204:</td>
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<tr>
<td>Computer Programming (27+3)</td>
<td>30</td>
</tr>
<tr>
<td>Continuum Mechanics-I (18+2)</td>
<td>20</td>
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<tr>
<td>MCG205:</td>
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<tr>
<td>Computer Aided Numerical Practical</td>
<td>50</td>
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</table>
Semester III & IV Separated for Pure and Applied Stream

**SEMESTER-III: Pure Stream**
**Duration: 6 Months**

<table>
<thead>
<tr>
<th>Total Marks: 250</th>
<th>Total No. of Lectures: 50 Hours per paper</th>
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Each Theoretical Paper Containing 45 Marks and 5 Marks for Internal Assessment

<table>
<thead>
<tr>
<th>Paper</th>
<th>Marks</th>
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<tbody>
<tr>
<td>MPG301: Modern Algebra-II (27+3)</td>
<td>30</td>
</tr>
<tr>
<td>General Topology-I (18+2)</td>
<td>20</td>
</tr>
<tr>
<td>MPG302: Graph Theory (36+4)</td>
<td>40</td>
</tr>
<tr>
<td>Set Theory-I (9+1)</td>
<td>10</td>
</tr>
<tr>
<td>MPG303: Set Theory –II &amp; Mathematical Logic (27+3)</td>
<td>30</td>
</tr>
<tr>
<td>Functional Analysis-II (18+2)</td>
<td>20</td>
</tr>
<tr>
<td>MPS304: Special Paper-I (45+5)</td>
<td>50</td>
</tr>
<tr>
<td>MPS305: Special Paper-II (45+5)</td>
<td>50</td>
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**SEMESTER-IV: Pure Stream**
**Duration: 6 Months**

<table>
<thead>
<tr>
<th>Total Marks: 250</th>
<th>Total No. of Lectures: 50 Hours per paper</th>
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Each Theoretical Paper Containing 45 Marks and 5 Marks for Internal Assessment

<table>
<thead>
<tr>
<th>Paper</th>
<th>Marks</th>
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<tbody>
<tr>
<td>MPG401: Modern Algebra-III (45+5)</td>
<td>50</td>
</tr>
<tr>
<td>MPG402: General Topology-II (27+3)</td>
<td>30</td>
</tr>
<tr>
<td>Functional Analysis-III (18+2)</td>
<td>20</td>
</tr>
<tr>
<td>MPS403: Special Paper-III (45+5)</td>
<td>50</td>
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<tr>
<td>MPS404: Special Paper-IV (45+5)</td>
<td>50</td>
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<tr>
<td>MPT405: Term Paper (30+15+5)</td>
<td>50</td>
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</table>

Term Paper MPT405 is related with the Special papers and the Marks distribution is 30 Marks for written submission and 15 Marks for Seminar Presentation and 5 Marks for Viva-Voce.
**SEMESTER-III: Applied Stream**

**Duration:** 6 Months  
**Total Marks:** 300  
**Total No. of Lectures:** 50 Hours per paper  

Each Theoretical Paper Containing 45 Marks and 5 Marks for Internal Assessment

<table>
<thead>
<tr>
<th>Paper</th>
<th>Marks</th>
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</thead>
<tbody>
<tr>
<td>MAG301: Methods of Applied Mathematics -I (45+5)</td>
<td>50</td>
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</tbody>
</table>
| MAG302: Methods of Applied Mathematics -II (27+3)  
Theory of Electro Magnetic Fields (18+2) | 30  
|                        | 20    |
| MAG303: Continuum Mechanics-II (27+3)  
Dynamical Systems (18+2) | 30  
|                        | 20    |
| MAS304: Special Paper-I (45+5) | 50    |
| MAS305: Special Paper-II (45+5) | 50    |

**SEMESTER-IV: Applied Stream**

**Duration:** 6 Months  
**Total Marks:** 250  
**Total No. of Lectures:** 50 Hours per paper  

Each Theoretical Paper Containing 45 Marks and 5 Marks for Internal Assessment

<table>
<thead>
<tr>
<th>Paper</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAG401: Continuum Mechanics-III (45+5)</td>
<td>50</td>
</tr>
</tbody>
</table>
| MAG402: Elements of Quantum Mechanics (27+3)  
Chaos and Fractals (18+2) | 30  
|                        | 20    |
| MAS403: Special Paper-III (45+5) | 50    |
| MAS404: Special Paper-IV (45+5) | 50    |
| MAT405: Term Paper (30+15+5) | 50    |

Term Paper MAT405 is related with the Special papers and the Marks distribution is 30 Marks for written submission and 15 Marks for Seminar Presentation and 5 Marks for Viva-Voce.
Cluster of Special Paper-I & III For Semester-III & IV respectively each of 50 marks
(Pure Stream):
A  Differential Geometry of Manifolds-I & II
B  Advanced Real Analysis-I & II
C  Advanced Functional Analysis-I & II
D  Rings of Continuous Function-I & II
E  Theory of Rings and Algebra-I & II
F  Non-linear Optimization in Banach Spaces-I & II
G  Harmonic Analysis-I & II
H  Applied Functional Analysis-I & II

Cluster of Special Paper-II & IV For Semester-III & IV respectively each of 50 marks
(Pure Stream):
A  Measure and Integration- I & II
B  Operator Theory and Applications- I & II
C  Algebraic Topology- I & II
D  Lattice Theory-I & II
E  Advanced Operations Research- I & II
F  Geometric Functional Analysis- I & II
G  Proximities, Nearnesses and Extensions of Topological Spaces- I & II
H  Advanced Complex Analysis- I & II

Cluster of Special Paper-I & III For Semester–III & IV respectively each of 50 marks
(Applied Stream):
A  Viscous Flows, Boundary Layer Theory and Magneto Hydrodynamics-I & II
B  Elasticity-I & II
C  Elasticity and Theoretical Seismology-I & II
D  Applied Functional Analysis -I & II

Cluster of Special Paper-II & IV For Semester–II & IV respectively each of 50 marks
(Applied Stream):
A  Quantum Mechanics- I & II
B  Advanced Operations Research- I & II
C  Inviscid Compressible Flows and Turbulence- I & II
* All the students will have to take the Special paper-I & III for Semester-III & IV and Special Paper-II & IV for Semester-III & IV respectively from the same topic, and the Special paper-I & II and Special paper-III & IV from the respective clusters of special papers for Semester-III and Semester-IV respectively. The clusters of special papers to be offered in a particular year shall be decided by the Department. Students of Applied Mathematics stream may also opt *Applied Functional Analysis-I & II* as the Special paper-I and Special paper-III respectively, if they so desire. Students of Pure Mathematics Stream may also opt the *Advanced Operations Research-I & II* as the Special paper-II and Special paper-IV respectively, if they so desire.

**DETAILED SYLLABUS**

**SEMESTER-I**

*Paper – MCG101*

*(Functional Analysis-I & Real Analysis-I)*

**Unit-I**

**Functional Analysis-I**

*Total Lectures: 40 (Marks – 30)*

- Baire category theorem. Normed linear spaces, continuity of norm function, Banach spaces, Spaces $\mathbb{C}^n$, $C[a,b]$ (with supmetric), $c_0$, $l_p$ ($1 \leq p \leq \infty$) etc; (10L)
- Linear operator, boundedness and continuity, examples of bounded and unbounded linear operators. (10L)
- Banach contracton Principle – application to Picard’s existence theorem and Implicit function theorem. (8L)
- Inner product, Hilbert spaces, examples such as $l_2$ spaces, $L_2[a,b]$ etc; C-S inequality, Parallelogram law, Pythagorean law, Minkowski inequality, continuity and derivatives of functions from $\mathbb{R}^n$ to $\mathbb{R}^n$. (12L)
Unit-2
Real Analysis-I

Total Lectures : 25
(Marks – 20)

Monotone functions and their discontinuities, Functions of bounded variation on an interval, their properties, Riemann-Stieltjes integral, existence, convergence problem and other properties. (12L)

Lebesgue outer measure, countable subadditivity, measurable sets and their properties, Lebesgue measure, measurable functions, equivalent functions, continuity and measurability, monotonocity and measurability, operation on collection of measurable functions, pointwise limit of a sequence of measurable functions, measurability of Supremum and Infimum, simple function and measurable function. (13L)

References

2. Lusternik and Sovolev- *Functional Analysis*
5. Vulikh- *Functional Analysis*
15. Royden – Real Analysis, PHI, 1989
Vector spaces, Euclidean space, Unitary space, orthonormal basis, Gram-Schmidt orthogonalization process. (8L)
Linear transformation in finite dimensional spaces, matrix of linear, rank and nullity, annihilator of a subset of a vector space. (5L)
Eigen vectors, spaces spanned by eigen vectors, similar and congruent matrices, characteristic polynomial, minimal polynomial, diagonalization, diagonalization of symmetric and Hermitian matrices, Cayley-Hamilton theorem, reduction of a matrix to normal form, Jordan Canonical form. (17L)
Quadratic form, Reduction to normal form, Sylvester’s law of inertia, simultaneous reduction of two quadratic forms, applications to Geometry & Mechanics. (10L)
Groups: Homomorphism, Isomorphism of groups, First and second isomorphism theorems, automorphisms and automorphism group, Inner automorphisms, groups of order 4 and 6, Normal sub groups and correspondence theorem for groups, simple groups. (10L)

Rings: Ring, commutative rings with identity, Prime & irreducible elements, division ring, Quaternions, idempotent element, Boolean ring, ideals, Prime ideal, maximal ideal, Isomorphism theorems, relation between Prime and maximal ideal, Euclidean domain, Principal ideal domain, Unique factorization domain, Polynomial rings. (15L)

References:
6. P. M. Cohn – *Basic Algebra*.
7. S. Lang – *Algebra*.
8. S. Lang – *Linear Algebra*.

Paper – MCG103

*(Elements of General Topology & Complex Analysis-I)*

**Unit-I**

**Elements of General Topology**

Total Lectures : 40 (Marks – 30)

Topological spaces; definition, open sets, closed sets, closure, denseness, neighbourhood, interior points, limit points, derived sets, basis, subbasis, subspace. (10L)

Alternative way of defining a topology using Kuratowski closure operators and neighbourhood systems. (5L)

Continuous functions, homeomorphism and topological invariants. (3L)
First and second countable spaces, Lindelöf spaces, separable spaces and their relationship. (8L)
Separation axioms: $T_0$, $T_1$, $T_2$, $T_3$, $T_{3\frac{1}{2}}$, $T_4$ spaces, their simple properties and their relationship. (8L)
Introduction to connectedness and compactness. (6L)

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**Unit-2**

**Complex Analysis -I**

Total Lectures : 25 (Marks – 20)

Complex Integration, line integral and its fundamental properties, Cauchy’s fundamental theorem, Cauchy’s integral formula and higher derivatives, power series expansion of analytic functions. (14L)

Zeros of analytic functions and their limit points, entire functions, Liouville’s theorem. Fundamental theorem of algebra. (6L)

Simply connected region and primitives of analytic functions, Morera’s theorem. (5L)

**References**

1. Simmons – *Introduction to Topology & Modern Analysis*
2. Munkresh – *Topology*
3. W. J. Thron – *Topological Structures*
4. Joshi – *General Topology*
5. J. L. Kelley – *General Topology*
6. J. B. Conway – *Functions of one Complex Variable* [Narosa]
7. R. B. Ash – *Complex Variable* [A.P.]
8. Punoswamy – *Functions of Complex Variable*
9. Gupta & Gupta – *Complex Variable*
10. W. Churchill- *Theory of Functional of Complex variable*
11. E. T. Copson- Functions of Complex variable
12. Philips- Functions of Complex variable
Paper – MCG104

(Ordinary Differential Equations & Special Functions, Operations Research-I)

Unit-1

Ordinary Differential Equations & Special Functions

Total Lectures : 40  (Marks – 30)

Ordinary Differential Equations

First order system of equations: Well-posed problems, existence and uniqueness of the solution, simple illustrations. Peano’s and Picard’s theorems (statements only) (8L)
Linear systems, non-linear autonomous system, phase plane analysis, critical points, stability, Linearization, Liapunov stability, undamped pendulum, Applications to biological system and ecological system (12L).

Special Functions

Series Solution : Ordinary point and singularity of a second order linear differential equation in the complex plane; Fuch’s theorem, solution about an ordinary point, solution of Hermite equation as an example; Regular singularity, Frobenius’ method – solution about a regular singularity, solutions of hypergeometric, Legendre, Laguerre and Bessel’s equation as examples.(10L)
Legendre polynomial : its generating function; Rodrigue’s formula, recurrence relations and differential equations satisfied by it; Its orthogonality, expansion of a function in a series of Legendre Polynomials.(6L)
Adjoint equation of n-the order: Lagrange’s identity, solution of equation from the solution of its adjoint equation, self-adjoint equation, Green’s function.(4L)

Unit-2

Operations Research-I

Total Lectures : 25  (Marks – 20)

Fundamental theorem of L.P.P. along with the geometry in n-dimensional Euclidean space (hyperplane, separating and supporting plane).(3L)
Standard forms of revised simplex method, Computational procedure, Comparison of simplex method and revised simplex method, Sensitivity analysis, Bounded variable method, The Primal Dual Method.(14L)
Mathematical formulation of Assignment Problem, Optimality condition, Hungarian method, Maximization case in Assignment problem, Unbalanced Assignment problem, Restriction on Assignment, Travelling salesman problem.(5L)

References :

2. Sasievir, Yaspan, Friedman – Operations Research: Methods and Problems (JW)
4. Taha – Operations Research
5. Schaum’s Outline Series – Operations Research
7. Swarup, Gupta & Manmohan – Operations Research
11. G. F. Simmons - Differential Equations [TMH]
13. E. D. Rainville – Special Functions [ Macmillan]
15. N. N. Lebedev - Special Functions and Their Applications [PH]

Paper – MCG105

(Principle of Mechanics-I & Numerical Analysis)

Unit-1

Principle of Mechanics-I

Total Lectures : 40 (Marks – 30)
Generalised co-ordinates: Degrees of freedom, Constraint, Principle of Virtual Work. Lagrangian formulation of Dynamics: Lagrange’s equations of motion for holonomic and non-holonomic systems. Ignoration of coordinates. (10L)


Configuration space and system point. Hamilton’s principle; Hamilton’s canonical equations of motion. Principle of energy. (10L)

Principle of least action, Canonical Transformations, Poisson Bracket. (10L)

Unit-2

Numerical Analysis

Total Lectures : 25

(Marks – 20)

Numerical Methods : Algorithm and Numerical stability. (2L)

Graffae’s root squaring method and Bairstow’s method for the determination of the roots of a real polynomial equation. (4L)

Polynomial Approximation : Polynomial interpolation; Errors and minimizing errors; Tchebyshev polynomials; Piece-wise polynomial approximation. Cubic splines; Best uniform approximations, simple examples. (4L)

Richardson extrapolation and Romberg’s integration method; Gauss’ theory of quadrature. Evaluation of singular integral. (4L)

Operators and their inter-relationships : Shift, Forward, Backward, Central differences; Averaging operators, Differential operators and differential coefficients. (2L)

Initial Value Problems for First and Second order O.D.E. by

(i) 4th order R – K method
(ii) RKF4- method
(iii) Predictor – Corrector method by Adam-Bashforth, Adam-Moulton and Milne’s method. (3L)

Boundary value and Eigen-value problems for second order O.D.E. by finite difference method and shooting method. (3L)

Elliptic, parabolic and hyperbolic P.D.E. (for two independent variables) by finite difference method; Concept of error, convergence & numerical stability. (3L)

References:

13
1. F. Chorlton – *A Text Book of Dynamic*
2. Synge and Griffith – *Principles of Mechanics*
3. D. T. Green Wood – *Classical Dynamics*
4. E. T. Whittaker – *A Treatise on the Analytical Dynamics of Particles and Rigid Bodies*
5. K. C. Gupta – *Classical Mechanics of Particles and Rigid Bodies*
6. F. Gantmacher – *Lectures in Analytical Mechanics*
7. H. Goldstein – *Classical Mechanics*
8. F. B. Hildebrand – *Introduction to Numerical Analysis*
9. Demidovitch and Maron – *Computational Mathematics*
10. F. Scheid – *Computers and Programming* (Schaum’s series)
13. A. Gupta and S. C. Basu – *Numerical Analysis*
14. Scarborough – *Numerical Analysis*
15. Atkinson – *Numerical Analysis*
SEMESTER-II

Paper – MCG201
(Complex Analysis-II & Real Analysis-II)

Unit-1

Complex Analysis-II

Total Lectures : 40 (Marks – 30)

Open mapping theorem.(5L)
Singularities. Laurent’s series expansion and classification of isolated singularities, essential
singularities and Casorati-Weierstrass’s theorem. Cauchy’s residue theorem and evaluation
of improper integrals.(12L)
Argument principle, Rouche’s theorem and its application.(5L)
Maximum modulus theorem.(3L)
Conformal mappings, Schwarz’s Lemma and its consequence.(10L)
Introduction to Analytic continuation.(5L)

Unit-2

Real Analysis-II

Total Lectures : 25 (Marks – 20)

Lebesgue integral of a simple function, Lebesgue integral of a non-negative (bounded or
unbounded) measurable function, Integrable functions and their simple properties, Lebesgue
integral of functions of arbitrary sign, Integrable functions, basic properties of the integral,
Integral of point wise limit of sequence of measurable functions- Monotone convergence
theorem and its consequences, Fatou’s lemma, Lebesgue dominated convergence theorem.
Comparison of Lebesgue’s integral and Riemann integral, Lebesgue criterion of Riemannian
integrability. (17L)
Fourier series, Dirichlet’s kernel, Riemann-Lebesgue theorem, Pointwise convergence of
Fourier series of functions of bounded variation. (8L)

1. J. B. Conway – Functions of one Complex Variable [Narosa]
2. R. B. Ash – Complex Variable [A.P.]
3. Punoswamy – Functions of Complex Variable
4. Gupta & Gupta – Complex Variable
6. C. Goffman – Real Functions
7. Burkil & Burkil – Theory of Functions of a Real Variable
8. Goldberg – Real Analysis
9. Royden – Real Analysis
10. Lahiri & Roy – Theory of Functions of a Real Variable
11. Apostol – Real Analysis
12. Titchmarsh – Theory of Functions
13. Charles Scwarz-Measure, Integration and Functions Spaces.

**Paper – MCG202**

*(Partial Differential Equations & Differential Geometry)*

**Unit-1**

**Partial Differential Equations**

Total Lectures : 40  
(Marks – 30)

*General solution and complete integral of a partial differential equation; Singular solution;  
Integral surface passing through a curve and circumscribing a surface.(4L)  
First order P.D.E, : Characteristics of a linear first order P.D.E.; Cauchy’s problem;  
Solution of non-linear first order P.D.E. by Cauchy’s method of characteristics; Charpit’s  
method (application only).(8L)  
Second order linear P.D.E. : Classification, reduction to normal form; Solution of equations  
with constant coefficients by (i) factorization of operators, (ii) separation of variables;  
Solution of one-dimensional wave equation and diffusion equation; Solution of Laplace  
equation in Cylindrical and spherical polar co-ordinates. Formulation of Initial and  
Boundary Value Problem of P.D.E; Solution of Dirichlet’s and Numann’s problem of  
Laplace’s equation for a circle.(28L)*

**Unit-2**

**Differential Geometry**

Total Lectures : 25  
(Marks – 20)
Reciprocal base system, Intrinsic derivative, Parallel vector field along a curve Space Curve, Serret – Frenet formula. (8L)
Metric tensor of the surface, angle between two curves lying on the surface, parallel vector field on a surface, Geodesics on a surface, Its differential equation, Geodesic curvature of a surface curve, Tensor derivative. (10L)
First fundamental form of the surface, Gauss’s formula and second fundamental form of the surface, Meusnier theorem and Euler’s theorem. (7L)

References:
1. T. Amarnath – Partial Differential Equation
2. I. N. Sneddon – Partial Differential Equation
3. H. Goldstein – Classical Mechanics
5. C. E. Weatherburn – Differential Geometry
6. M. Postrikov – Lectures in Geometry, Linear Algebra and Differential Geometry
8. M. P. Do Carmo- Differential Geometry of Curves and Surfaces
9. B. O’Neill- Elementary Differential Geometry
10. Rutter- Geometry of Curves
11. Andrew Pressely- Elementary Differential Geometry

Paper – MCG203
(Operations Research-II & Principle of Mechanics-II)

Unit-I

Operations Research-II

Total Lectures : 40 (Marks – 30)
Deterministic Inventory control Models: Introduction, Classification of Inventories, Advantage of Carrying Inventory, Features of Inventory System, Deterministic inventory models including price breaks.(14L)

Standard form of Integer Programming, The concept of cutting plane, Gomory’s all integer cutting plane method, Gomory’s mixed integer method, Branch and Bound method.(10L)

Processing of n jobs through two machines, The Algorithm, Processing of n jobs through m machines, Processing of two jobs through m machines.(6L)

Project scheduling by PERT/CPM : Introduction, Basic differences between PERT and CPM, Steps of PERT/CPM Techniques, PERT/CPM network Components and Precedence Relationships, Critical Path analysis, Probability in PERT analysis, Project Crashing, Time cost Trade-off procedure, Updating of the Project, Resource Allocation.(10L)

Unit-2

Principle of Mechanics-II

Total Lectures : 25 (Marks – 20)

Theory of small oscillations. Normal co-ordinates. Euler’s dynamical equations of motion of a rigid body about a fixed point. Torque free motion. Motion of a top on a perfectly rough floor. Stability of top motion. Motion of a particle relative to rotating earth. Foucault’s pendulum.(20L)

Special Theory of Relativity : Postulates; Special Lorentz Transformation ; Fitz-Gerald contraction and time-dilation. Einstein’s velocity addition theorem. Relativistic mechanics of a particle, Energy equation $E = mc^2$. (5L)

References :

1. F. Chorlton – A Text Book of Dynamic
2. Synge and Griffith – Principles of Mechanics
3. D. T. Green Wood – Classical Dynamics
4. E. T. Whittaker – A Treatise on the Analytical Dynamics of Particles and Rigid Bodies
5. K. C. Gupta – Classical Mechanics of Particles and Rigid Bodies
6. I. S. Sokolnikoff – Mathematical Theory of Elasticity
7. Merovitch- A treatise on dynamics
Unit-1

Computer Programming

Total Lectures : 40 (Marks – 30)

Structured Programming in FORTRAN – 77: Subscripted variables, Type declaration, DIMENSION, DATA, COMMON, EQUIVALENCE, EXTERNAL statements. Function and subroutine sub – programs; Programs in FORTRAN – 77 (12L)

Programming in C: Introduction, Basic structures, Character set, Keywords, Identifiers, Constants, Variable-type declaration, Operators : Arithmetic, Relational, Logical, assignment, Increment, decrement, Conditional. (13L)

Operator precedence and associativity, Arithmetic expression, Evaluation and type conversion, Character reading and writing, Formatted input and output, Decision making (branching and looping) – Simple and nested IF, IF – ELSE, WHILE – DO, FOR. Arrays-one and two dimension, String handling with arrays – reading and writing, Concatenation, Comparison, String handling function, User defined functions. (15L)

Unit-2

Continuum Mechanics-I

Total Lectures : 40 (Marks – 30)

Continuous media, Deformation, Lagrangian and Eulerian approach: Analysis of strain (infinitesimal theory): (10L)
Analysis of stress; Invariants of stress and strain tensors. Principle of conservation of mass; Principle of balance of linear and angular momentum; Stress equation of motion. (20L)

Necessity of constitutive equations. Hooke’s law of elasticity, displacement equation of motion. Newton’s law of viscosity (statement only). (10L)

References:
1. Ram Kumar – Programming With Fortran – 77
2. P. S. Grover - Fortran – 77/90
5. Xavier, C. – C Language and Numerical Methods, (New Age International (P) Ltd. Pub.)
8. F. Scheid – Computers and Programming (Schaum’s series)
9. T. J. Chang – Continuum Mechanics (Prentice – Hall)
10. Truesdell – Continuum Mechanics (Schaum Series)
11. Mollar – Theory of Relativity
12. F. Gantmacher – Lectures in Analytical Mechanics
14. R. N. Chatterjee – Continuum Mechanics
15. H. Goldstein – Classical Mechanics

Paper – MCGP205

Computer Aided Numerical Practical

Total Practical Classes : 65 (Marks – 50)

(Numerical Practical Using FORTRAN – 77)
(Viva to be conducted on Paper – MG205 & MG206 only)

1. Inversion of a non-singular square matrix. (6L)
2. Solution of a system of linear equations by Gauss – Seidel method. (4L)
3. Integration by Romberg’s method. (5L)
4. Initial Value problems for first and second order O.D.E. by
   (i) Milne’s method (First order) (6L)
   (ii) 4th order Runge – Kutta method (Second order) (5L)
5. Dominant Eigen – pair of a real matrix by power method (largest and least). (14L)
6. B.V.P. for second order O.D.E. by finite difference method and Shooting method. (5L)
7. Parabolic equation (in two variables) by two layer explicit formula and Crank–Nickolson – implicit formula. (12L)
8. Solution of one dimensional wave equation by finite difference method. (8L)

References :
1. Ram Kumar – Programming With Fortran –77
2. P. S. Grover - Fortran – 77/90
5. Xavier, C. – C Language and Numerical Methods, (New Age International (P) Ltd. Pub.)

SEMESTER-III
PURE MATHEMATICS STREAM
Paper – MPG301
(Modern Algebra-II & Set Theory-I)

Unit-1
Modern Algebra-II
Total Lectures : 40 (Marks – 30)

Groups : Direct product (internal and external), Group action on a set, Conjugacy classes and conjugacy class equation, p-groups, Cauchy’s theorem, converse of Lagrange’s theorem for finite commutative groups, Syllow theorems and applications, Normal series, solvable
series, solvable and Nilpotent groups, Jördan-Holder Theorem, Finitely generated Abelian groups, Free Abelian groups. (20L)

**Rings :** Unique factorization domain; Factorization of polynomials over a field; Maximal, Prime and primary ideal; Noetherian and Artinian Rings; Hilbert basis theorem. (20L)

**References :**

9. S. Lang – Algebra.

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**Unit-2**

**General Topology-I**

**Total Lectures : 25** (Marks – 20)

Normal spaces, Urysohn’s lemma and Tietze’s extension theorem. (5L)

Product spaces, embedding lemma, Tychonoff spaces and characterization of Tychonoff spaces as subspaces of cubes. (6L)

Nets, filters subnets and convergence (4L)

Compactness, Compactness and continuity, countable compactness, sequential compactness, BW compactness and their relationship, Local compactness, Tychonoff theorem (on Product of Compact Spaces) (10L)

**References :**

1. J. Dugundji – *Topology* (Allayn and Bacon, 1966)

**Paper – MPG302**

*(Graph Theory & Set Theory-I)*

**Unit-1**

*Graph Theory*

**Total Lectures : 55** (Marks – 40)

Graph, Subgraph, Complement, Isomorphism, Walks, Paths, cycles, connected components, Cut vertices and cut edges, Distance, radius and center, Diameter, Degree sequence, Havel-Hakimi Theorem (Statement only) (10L)

Trees, Centres of trees, Spanning trees, Eulerian and Semi Eulerian Graphs. Hamiltonian Graphs, Travelling Salesman Problem. (10L)

Vertex and edge connectivities, Blocks, Mengers Theorem. Clique Number, Independence number, Matching number, Vertex and edge conserving number, domination number, Ramsay’s Theorem. (8L)

Chromatic number, Bipartite graph. Broke’s Theorem, Mycielski Construction, Chromatic polynomial, edge colouring number, König Theorem. (6L)

Adjacency matrix, Incidence matrix, Cycle rank and co-cycle rank, Fundamental Cycles with respect to Spanning tree and Cayley’s theorem on trees. (5L)

Planar graphs, Statement of Kuratowski Theorem, Isomorphism properties of graphs, Eulers formula, 5 colour theorem. Statement of 4 colour theorem, Dual of a planar Graph. (8L)

Directed Graph, Binary relations, directed paths, fundamental Circuits in Digraphs, Adjacency matrix of a Digraph. (8L)

**References:**

2. Nar Sing Deo – *Graph Theory* (Prentice-Hall, 1974)
3. F. Harary – *Graph Theory* (Addison-Wesley, 1969)

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**Unit-2**

**Set Theory-I**

**Total Lectures : 15**

(Marks – 10)

Axiom of choice, Zorn’s Lemma, Hausdorff maximality principle, Well-ordering theorem and their equivalence, General Cartesian product of sets, Cardinal numbers and their ordering, Schröder-Bernstein theorem. (15L)

**References :**

1. K. Kuratowski – *Introduction to Set Theory and Topology*
2. E. Mendelson – *Introduction to Mathematical Logic*
3. R. R. Stoll – *Set Theory and Logic*
4. I. M. Copi – *Symbolic Logic*
5. W. Sierpienski – *Cardinal and Ordinal Numbers*

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**Paper – MPG303**

(*Set Theory-II & Mathematical Logic, Functional Analysis-II*)

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**Unit-1**

**Set Theory-II & Mathematical Logic**

**Total Lectures : 40**

(Marks – 30)

**Set Theory-II**

Addition, multiplication and exponentiation of cardinal numbers, the cardinal numbers $\aleph_0$ and $\mathbb{C}$ and their relation. (8L)

Totally ordered sets, order type, well-ordered sets, ordinal numbers, initial segments, ordering of ordinal numbers, addition and multiplication of ordinal numbers, sets of ordinal numbers, Transfinite induction. (7L)

**Mathematical Logic**
Statement calculus: Propositional connectives, statement form, truth functions, truth tables, tautologies, contradiction, adequate sets of connectives (10L)

Arguments: Proving validity rule of conditional proof. Formal statement calculus, formal axiomatic theory L, Deduction theorem (8L)

Consequences. Quantifiers, universal and existential; symbolizing everyday language. (7L)

References:
1. K. Kuratowski – Introduction to Set Theory and Topology
2. E. Mendelson – Introduction to Mathematical Logic
3. R. R. Stoll – Set Theory and Logic
4. I. M. Copi – Symbolic Logic
5. W. Sierpienski – Cardinal and Ordinal Numbers

Unit-2

Functional Analysis-II

Total Lectures: 25 (Marks – 20)

Completion of Metric space. Equicontinuous family of Functions. Compactness in C[0,1] (Arzela-Ascoli’s Theorem). Convex sets in linear spaces. (8L)


Principle of Uniform Boundedness (Banach-Steinaus), Open Mapping theorem. Closed graph theorem, Extension of continuous linear mapping. (7L)

References:
1. Lusternik and Sovolev-Functional Analysis
3. K.K. Jha- Functional Analysis, Student’s Friends, 1986
4. Vulikh- Functional Analysis
11. B.V. Limaye- Functional Analysis, Wiley Easten Ltd

Paper – MPS304
(Special Paper-I)

Total Lectures : 65 (Marks – 50)

A: Differential Geometry of Manifolds-I

Exterior algebra. Exterior derivative. (10L)
Topological groups. Lie groups and Lie algebras. Product of two Liegroups. One parameter subgroups and exponential maps. Examples of Liegroups. Homomorphism and Isomorphism. Lie transformation groups. General linear groups. (15L)

References:

**B: Advanced Real Analysis-I**

Upper and lower limits of real function and their properties. (5L)

**References:**

1. Hewitt and Stormberg – *Real and Abstract Analysis*
2. H. L. Royden – *Real Analysis*
3. Saks – *Theory of Integrals*
4. W. Rudin – *Real and Abstract Analysis*
5. M. E. Munroe – *Measure and Integration*

**C: Advanced Functional Analysis-I**

Topological vector spaces, Local base and its properties, Separation properties, Locally compact topological vector space and its dimension. Convex Hull and representation Theorem, Extreme points, Symmetric sets, Balanced sets, absorbing sets, Bounded sets in topological vector space. Linear operators over topological vector space, Boundedness and
continuity of Linear operators, Minkowski functionals, Hyperplanes, Separation of convex sets by Hyperplanes, Krein-Milman Theorem on extreme points. (30L)
Locally convex topological vector spaces, Criterion for normability, Seminorms, Generating family of seminorms in locally convex topological vector spaces. Barreled spaces and Bornological spaces, Criterion for Locally convex topological vector spaces to be (i) Barreled and (ii) Bornological. (15L)
Strict convexity and Uniform convexity of a Banach space. Uniform Convexity of a Hilbert Space. Reflexivity of a uniformly convex Banach space, Weierstrass approximation theorem in $C[a,b]$ (20L)

References :

5. Diestel- Application of Geometry of Banach Spaces
9. Lipschitz- General Topology, Schaum Series

D: Rings of Continuous Function-I
The ring $\mathbb{C}(X)$ of the real valued continuous function on a topological space $X$, its subrings, the subring $\mathbb{C}^*(X)$, their Lattice structure, ring homomorphisms and lattice homomorphism.(15L)
Zero-sets cozero-sets, their unions and intersection, completely separated sets, $\mathbb{C}^*$ - embedding, Urysohn’s extension theorem and C-embedding. Pseudocompactness and internal characterization of Pseudocompact spaces.(15L)
Ideals, Z-filters, maximal ideals, prime ideals, prime filters and their relation.(5L)
Completely regular spaces and the zero-sets, weak topologies determined by $\mathbb{C}(X)$ and $\mathbb{C}^*(X)$. Stone-Čech’s therem concerning adequacy of Tychonoff spaces $X$ for investigation of $\mathbb{C}(X)$ and $\mathbb{C}^*(X)$, compact subsets and C – embedding, locally compact spaces and their properties.(10L)
Convergence of Z – filters, cluster points, prime Z – filters and convergence and fixed Z-filters.(5L)
Fixed ideals and compactness, fixed maximal ideals of $\mathbb{C}(X)$ and $\mathbb{C}^*(X)$, their characterizations, the residue class rings modulo fixed maximal ideals in $\mathbb{C}(X)$ and $\mathbb{C}^*(X)$ and the field of reals. Relation between fixed maximal ideals in $\mathbb{C}(X)$ and $\mathbb{C}^*(X)$. Compactness and fixed ideals.(10L)
P- spaces, P-points and their properties, characterization of P-spaces, properties of P-spaces.(5L)

References :


E: Theory of Rings And Algebras-I

Rings and Ideals: Definitions, Ideal. Quotient rings and homorphisms, The field of quotients, Minimal and maximal conditions, Primary decomposition, Polynomial rings. (15L)
Modules: Preliminaries, Direct sums and free modules, projective modules, Tensor products, field and matrix representations, Algebras. (20L)

The Jacobson Radical: Primitive rings, The Density Theorem, Structure theorems. (10L)

References:
5. T. Y. Lam – *Noncommutative Rings* (Springer-Verlag)
6. N. Jacobson – *Basic Algebra –II*
7. I. N. Herstein – *Noncommutative Rings*
8. N. J. Divisky – *Rings and Radicals*
10. M. R. Adhikari – Groups, Rings, Modules and applications

**F: Non Linear Optimization In Banach Spaces-I**

Review of Weak Convergence in normed spaces, reflexivity of Banach spaces, Hahn-Banach theorem and partially ordered linear spaces. (20L)

Existence Theorems for Minimal Points –Problem formulation. Existence theorems. Set of minimal points. (15L)

Applications to approximations and optimal control problems. (15L)


References:
G: Harmonic Analysis-I

Basic properties of topological groups, subgroups, quotient groups and connected groups. Discussion of Haar Measure without proof on R, T, Z, and some simple matrix groups. $L^1(G)$ and convolution with special emphasis on $L^1(\mathbb{R})$, $L^1(T)$, $L^1(Z)$. Approximate identities. (20L)

Fourier series. Fejer’s theorem. The classical kernels. Fejer’s Poisson’s and Dirichlet’s summability in norm and point wise summability. Fatou’s Theorem. The inequalities of Hausdorff and Young. (20L)


References:
5. R. R. Goldberg – Fourier transforms
6. T. Huissain – Introduction to topological groups

H: Applied Functional Analysis-I


Multilinear forms. Analyticity Theorems. Non-linear Volterra operators. (15L)

References:
A: Measure and Integration-I


References:

1. I. P. Rana – Measure and Integration
2. G. D. Barra – Measure and Integration
3. Hewitt and Stormberg – Real and Abstract Analysis
4. H. L. Royden – Real Analysis
5. Saks – Theory of Integrals
6. W. Rudin – Real and Abstract Analysis
7. M. E. Munroe – Measure and Integration
8. Taylor - Measure and Integration
B: Operator Theory And Applications-I

Adjoint operators over normed linear spaces; their algebraic properties. Compact operators on normal linear spaces, Sequence of Compact operators, Compact extensions, Weakly compact-operators(10L) operator equation involving compact operators, Fredholm alternative; Adjoint operators on Hilbert-spaces, Self-adjoint operators, their algebraic properties; Unitary operators, normal operators in Hilbert spaces, positive operators, their-sum, product; Monotone sequence of positive operators, square-root of positive operator, Projection operators. (20L) Their sum and product; Idempotent operators, positivity norms of Projection operators; Limit of monotone increasing sequence of Projection operators. (35L)

References:
11. Vulikh- Functional Analysis
13. Lipschitz-General Topology, Schaum Series

C: Algebraic Topology-I

Homotopy : Definition and some examples of homotopies, homotopy type and homotopy equivalent spaces, retraction and deformation, H-space.
Category: Definitions and some examples of category, factor and natural transformation. (10L)

Fundamental group and covering spaces: Definition of the fundamental group of a space, the effect of a continuous mapping on the fundamental group, fundamental group of a product space, notion of covering spaces, liftings of paths to a covering space, fundamental groups of a circle. (20L)

Universal cover, its existence, calculation of fundamental groups using covering space. Projection space and torus, homomorphisms and automorphisms of covering spaces, deck transformation group, Borsuk–Ulam theorem for $S^2$, Brouwer fixed-point theorem in dimension 2. (35L)

**References:**

1. W. S. Massey – *Algebraic Topology*
2. W. S. Massey – *Singular Homology Theory*
3. E. H. Spanier – *Algebraic Topology*
4. B. Gray – *Homotopy Theory An Introduction to Algebraic Topology*
5. C. R. Bredon – *Geometry and Topology*

**D: Lattice Theory-I**

Introduction: Partially ordered sets, graphs, order isomorphism, Maximal minimal condition, Jordan-Dedekind chain condition, dimension function. (10L)

Definition of an algebra, Lattices as algebras, density principle, Lattices as partially ordered sets sublattices Ideals, complements, semicomplements, atoms, Irreducible and prime elements, Morphisms homomorphisms, ideals direct products. (20L)

Closure operation, Dedekind condition, Dedekind cuts. Completion, interval topology. (15L)

Distributive and modular lattices, modularity and distributivity criterion, distributive sublattices of modular lattices, transposed intervals, meet representation in modular and distributive lattices. (20L)

**References:**

1. G. Szasz – Lattice Theory
2. G. Birkhoff – Lattice Theory
3. B. H. Arnold – Logic and Boolean Algebra
Non-linear Optimization: Local and global minima and maxima, convex functions and their properties, Method of Lagrange multiplier. (8L)
Optimality in absence of differentiability, Slater constraint qualification, Karlin’s constraint qualification, Kuhn-Tuckers Saddle point optimality conditions, Optimality criterion with differentiability and convexity, separation theorems, Kuhn-Tuckers sufficient optimality theorem. (10L)
Unconstrained Optimization: Search method: Fibonacci search, Golden Section search; Gradient Methods: Steepest descent Quasi-Newton’s method, Davidon-Fletche-Reeves method, Conjugate direction method (Fletcher-Reeves method). (15L)
Optimality conditions: Kuhn-Tucker conditions – non negative constraints (6L)
Quadratic Programming: Wolfe’s Modified Simplex method, Beale’s method (8L)
Separable convex programming, Separable Programming Algorithm. (6L)
Network Flow: Max-flow min-cut theorem, Generalized Max flow min-cut theorem, Linear Programming interpretation of Max-flow min-cut theorem, Minimum cost flows, Minflow max-cut theorem. (12L)

References:
5. Luenberger – *Introduction to Linear and Non-Linear Programming*
6. S. Dano – *Non-Linear and Dynamic Programming*
12. M. A. Bhatti, Practical Optimization Methods, Springer -Verlag

**F: Geometric Functional Analysis-I**

Convexity and Topology. Weak Topology. (35L)

**References :**
1. Holmes – *Geometric Functional Analysis and Its Applications*

**G: Proximities, Nearnesses and Extensions of Topology Spaces-I**

Čech closure operator, closure spaces, symmetric Čech closure operator, continuity, homeomorphisms and their properties. Linkage compact topological spaces and their relation with compact topological spaces in presence of regularity condition. Extensions of closure spaces, trace system, principal (strict) extensions, ordering of extensions. Representation of principal $T_0$ extension of a $T_0$ topological space with a given trace system. The set of principal $T_0$ extensions of a $T_0$ topological spaces is a partially ordered set and its consequence for the class of $T_2$ compactification of a Tychonoff space. (35L)
(Basic) proximities, induced closure operators, proximity spaces, proximal neighbourhoods, $p$-continuous functions and their properties. Lattice structure of the basic proximities compatible with a symmetric closure space. Clans, clusters and relation between them. Basic proximities are clan generated structures. Classification of basic proximities: Riesz (RI-) proximities, Lodato (LO-) proximities, Efremovič (EF-) proximities, their characterization and relation between them. (30L)

**References :**

**H: Advanced Complex Analysis-I**

36
Analytic function, the functions $M(r)$ and $A(r)$. Theorem of Borel and Caratheodary, Convex function and Hadamard three-circle theorem, Phragmen-Lindelof theorem. (20L)
Harmonic function, Mean value property, Maximum principle, Harmonic function on a disk, Hamaek’s inequality, Dirichlet’s problem. (15L)
Integral function, Poisson Jenson formula, construction of an integral function with given zeros –Weierstrass theorem, Jensen’s inequality, order, exponent of convergence of zeros of an integral function, canonical product, genus, Hadamard’s factorization theorem, Borel’s theorems, Picard’s first and second theorems. (30L)

References:

1. J. B. Conway – Functions of One Complex Variable
2. L. V. Ahlfors – Complex Analysis
3. W. Rudin – Real and Complex Analysis
4. E. C. Titchmarsh – Theory of Functions
5. E. T. Copson – Function of a Complex Variable
6. R. P. Boas – Entire Functions
7. H. Cartan – Analytic Functions

SEMESTER-IV
PURE MATHEMATICS STREAM
Paper – MPG 401
(Modern Algebra-III)

Total Lectures : 65 (Marks – 50)
Field Theory: Extension of fields, Simple extensions, Algebraic and Transcendental extensions, Splitting fields, Algebraically closed fields, normal extension, separable extensions, Perfect field. (30L)

Automorphism of fields, Galois field, Galois extension, Fundamental Theorem of Galois theory, primitive elements, Solution of polynomial equations by radicals, Insolvability of the general equation of degree five or more by radicals, Cyclotomic extensions, Ordered field, Valuation, Completion. (20 L)

Modules: Artinian and Noetherian Modules, Fundamental Structure Theorem for finitely generated modules over a P.I.D. and its application to finitely generated Abelian groups. (15L)

References:

1. S. Lang – Algebra (P.H.I.)
2. Hungerford – Algebra.
5. S. Lang – Algebra.

Paper – MPG402

(General Topology-II & Functional Analysis-II)

Unit-1

General Topology-II

Total Lectures : 40 (Marks – 30)

Connectedness and characterization of connected subsets, union of connected subsets. Connected subsets of the real line, local connectedness, components, structure of open sets in locally connected second countable spaces, connectedness of the product spaces (10L) One-point Compactification, Stone-Čech compactification(without proof) (3L) Compactness in metric spaces, Properties of Compact metric spaces (4L) Urysohn’s metrization theorem, Uniform structure, uniform topology, uniform spaces, uniform continuity, Cauchy filter, total boundedness, completeness and compactness. (13L)
Homotopy of paths, covering spaces, fundamental group. Definition of the fundamental group of the circle. (10L)

References:
1. J. Dugundji – *Topology* (Allayn and Bacon, 1966)

Unit-2

Functional Analysis-II

Total Lectures : 25 (Marks – 20)


Properties of strong and weak convergence. Adjoint (Conjugate) operators and their properties. Hilbert Spaces, $L_p[a, b]$ $(1 \leq p \leq \infty)$. (5L)


References:
1. Lusternik and Sovolev- *Functional Analysis*
4. Vulikh- *Functional Analysis*

**Paper – MPS403**

*(Special Paper-III)*

**Total Lectures : 65**

(Marks – 50)

**A: Differential Geometry of Manifolds-II**


Almost Complex manifolds. Nijenhuis tensor. Contravariant and covariant almost analysis vector fields. F-connection. (15L)

**References :**

B: Advanced Real Analysis-II

Density of arbitrary linear sets. Lebesgue density theorem. Approximate continuity. Properties of approximately continuous functions. Bounded approximately continuous function over \([a,b]\) and exact derivative. (15L)
The Perron integral: Definitions and basic properties, Comparison with Lebesgue integral and Newton integral. (10L)

Sets of the 1st and of the 2nd categories. Baires theorem for \(G_\delta\), residual and perfect sets, points of condensatia of a set. (10L)
Baire classification of functions. Functions of Baire class one. Baire’s theorem. Semicontinuous functions. (15L)

References:
1. Hewitt and Stormberg – *Real and Abstract Analysis*
2. H. L. Royden – *Real Analysis*
3. Saks – *Theory of Integrals*
4. W. Rudin – *Real and Abstract Analysis*
5. M. E. Munroe – *Measure and Integration*

C: Advanced Functional Analysis-II

Stone-Weierstrass W-capital theorem in \(C(X,R)\) and \(C(X,C)\) where \(X\) is a compact Hausdorff space, Representation theorem for bounded linear functionals on \(C[a,b]\), \(L_p\) \((1 \leq p < \infty)\) and \(L_p[a,b]\), \((1 \leq p < \infty)\), consequences of uniform boundedness principle, weak topology, weak* topology, Banach-Alaoglu theorem. (25L)
Approximation Theory in Normal Linear space, Best approximation, Uniqueness Criterion, Separable Hilbert Space. (15L)
Banach Algebra, Identity element, analytic property of resolvent Operator, Compactness of Spectrum, Spectral radius and Spectral mapping Theorem for polynomials, Gelfand Theory on representation of Banach Algebra, Gelfand Neumark Theorem. (25L)
References:

5. Diestel- *Application of Geometry of Banach Spaces*
12. Holmes- *Geometric Functional Analysis and its Application*

D: Rings of Continuous Function-II

Partially ordered rings, convex ideals, absolutely convex ideals, properties of convex ideals, lattice ordered rings, total orderedness of the residue class rings modulo prime ideals in \( C(X) \) and \( C^*(X) \), real ideals, hyper-real ideals in \( C(X) \). Limit ordinal, non-limit ordinals, compactness of the spaces of the ordinals, first uncountable ordinals space and its “one point compactification” and relation between the rings of continuous function on them, Characterization of real ideals.(20L)
Cluster point and convergence of \( \mathbb{Z} \)-filters on a dense subset of a Tychonoff space. Characterization of \( \mathbb{C}^* \) - embedded dense subset of a Tychonoff space. Construction of Stone-Čech compactification. More specific properties of \( \beta N \) and \( \beta Q \) and \( \beta R \).(15L)
Characterization of maximal ideas in $C^*(X)$ and $C(X)$. Gelfand-Kolmogorov theorem. Structure space of a commutative ring - another description of $\beta X$. The Banach-Stone theorem.(15)

Partial ordered set $K(X)$ of the $T_2$ Compactifications of a Tychonoff space $X$, elements of $K(X)$ and the subsets of $C^*(X)$. Local compactness and the complete lattice $K(X)$.(15L)

References:

E: Theory of Rings And Algebras -II

Other Radicals and Radical Properties : The Levitski Radical, Brown-Meloy radicals, Amitsur’s properties, Relations among the radicals. (15L)

Generalizations of the notions of radicals to other systems : Algebras, Group Algebras, Near rings, Groups, Lattices. (10L)

Lie & Jordan Algebras : Definitions, Nilpotency and Solvability, A structure theorem for nonassociative algebras, Jordan Algebras, Lie Algebras, Simple Lie and Jordan algebras. (20L)

Category Theory : Definition, Functions, Objects and morphisms, Kernels and images, Exact Categories, Products & limits, abelian Categories.

Radical subcategories, Applications of sheaf theory to the study of rings.

Elements of Universal Algebra. (20L)

References:
5. T. Y. Lam – *Noncommutative Rings* (Springer-Verlag)
7. N. Herstein – *Noncommutative Rings*
F: Non Linear Optimization In Banach Spaces-II

Tangent Cones-Definition and properties. Optimality Conditions. Lyusternik theorem.
Generalized Lagrange Multiplier Rule – Problem formulation. Necessary and Sufficient
optimality conditions. Application to optimal control problems. (20L)
Application to approximation problems. (15L)
Some special optimization problems-Linear quadratic optimal control problems. Time
optimal control problems. (30L)

References:
3. A. V. Balakrishnan – Applied Functional Analysis (Springer-Verlag)

G: Harmonic Analysis-II

Hardy spaces on the unit circle. Invariant subspaces. Factoring. Proof of the F. and M. Riesz
theorem. Theorems of Beurings and Szego in multiplication operator form. Structure of inner
and outer functions. (20L)
The Inequalities of Hardy and Hilbert. Conjugate functions. Theorems of Kolmogorov &
Zygmund and M. Riesz & Zygmund on conjugate functions. (20L)
The conjugate function as a singular integral. Statement of theBurkholder-Gundy Silverstein
Theorem on T. Maximal functions of Hardy and Littlewood Translation. The Theorems of
Wiener and Beurling. The Titchmarsh Convolution Theorem. Wiener’s Tauberian Theorem.
Spectral sets of bounded functions. (25L)

References:
5. R. R. Goldberg – *Fourier transforms*
6. T. Huissain – *Introduction to topological groups*

**H: Applied Functional Analysis-II**


**References:**

2. Dunford and Schwartz-*Linear operators*, vol. 1 & 11.
3. S. G. Krein-*Linear Differential Equations in a Banach space.*

**Paper – MPS404**

*(Special Paper-IV)*

**Total Lectures : 65** (Marks – 50)

**A: Measure and Integration-II**

Measurable Rectangles, Elementary sets. Product measures. Fubini’s theorem. (20L)
L\textsubscript{p} [a,b] – spaces (1 \leq p \leq \infty). Holder and Minkowski inequality. Completeness and other properties of L\textsubscript{p} [a, b] spaces. Dense subspaces of L\textsubscript{p} [a, b] – spaces. Bounded linear functionals on L\textsubscript{p} [a, b] – spaces and their representations. (15L)

References:

1. I. P. Rana – *Measure and Integration*
2. G. D. Barra – *Measure and Integration*
3. Hewitt and Stormberg – *Real and Abstract Analysis*
4. H. L. Royden – *Real Analysis*
5. W. Rudin – *Real and Abstract Analysis*
6. M. E. Munroe – *Measure and Integration*
7. Taylor - *Measure and Integration*

B: Operator Theory And Applications-II

Spectral properties of bounded-Linear operators in normed linear space; Spectrum, regular value, resolvant of operator; closure property and boundedness property of spectrum, spectral radius. (20L)

Eigenvalues, eigen-vectors of self-adjoint operators in Hilbert space, Resolvant sets, real property of spectrum of self-adjoint operators, range of spectrum, Orthogonal direct sum of Hilbert space,(20L)

Spectral-theorem for compact normal operators, Sesquilinear functionals, property of boundedness and symmetry, Generalisation of Cauchy-Schwarz inequality. (15L)

Unbounded-operators and their adjoint in Hilbert spaces. (10L)

References:

11. Vulikh—*Functional Analysis*
13. Lipschitz—*General Topology*, Schaum Series

C: Algebraic Topology-II

Introduction of singular homology and cohomology group by Eilenberg and steenrod axioms.
Existence and Uniqueness of singular homology and cohomology theory. (20L)
Calculation of homology and cohomology groups for circle. Projective spaces, torus relation between $H_1(X)$ and $\pi_1(X)$. (20L)
Singular cohomology ring, calculation of cohomology ring for projective spaces. Fibre bundles: Definitions and examples of bundles and vector bundles and their morphisms, cross sections, fibre products, induced bundles and vector bundles and their morphisms, cross sections, fibre products, induced bundles and vector bundles, homotopy properties of vector bundles. Homology exact sequence of a fibre bundle. (25L)

References:

1. W. S. Massey—*Algebraic Topology*
2. W. S. Massey—*Singular Homology Theory*
3. E. H. Spanier—*Algebraic Topology*
4. B. Gray—*Homotopy Theory An Introduction to Algebraic Topology*
5. C. R. Bredon—*Geometry and Topology*

D: Lattice Theory-II

Covering condition in modular lattice, modular lattices of locally finite length, Complemented modular lattices, Boolean algebras, complete Boolean algebras, Boolean algebras and Boolean rings, valuation of a lattice, metric and quasimetric lattice. (25L)
Complete Lattice, conditionally complete Lattices, Fix point theorem, Compactly generated lattices, subalgebra lattices. (20L)
Birkhoff lattices, Semimodular lattices, Complemented semimodular lattices, Ideal chains, Ideal lattices, Distributive lattices and ring of sets, Congruence relations, Ideals and congruence relations. (20L)

References:
1. G. Szasz – Lattice Theory
2. G. Birkhoff – Lattice Theory
3. B. H. Arnold – Logic and Boolean Algebra

E: Advanced Operations Research-II

Dynamic Programming: Characteristics of Dynamic Programming problems, Bellman’s principle of optimality (Mathematical formulation)
Model – 1: Single additive constraint, multiplicative separable return,
Model – 2: Single additive constraint, additively separable return,
Model – 3: Single a multiplicative constraint, additively separable return,
Multistage decision process – Forward and Backward recursive approach, Dynamic Programming approach for solving linear and non-linear programming problems, Application – Single-item N-period deterministic inventory model. (25L)
Geometric Programming: Elementary properties of Geometric Programming and its applications. (8L)
Queuing Theory: Introduction, characteristic of Queuing systems, operating characteristics of Queuing system. Probability distribution in Queuing systems. Classification of Queuing models. Poisson and non-Poisson queuing models (32L)

References:
2. S. Dano – Non-Linear and Dynamic Programming
Extreme points. Convex functions and optimization. Some More Applications: The category Theorems. (30L)

References:
1. Holmes – Geometric Functional Analysis and Its Applications

G: Proximities, Nearnesses and Extensions of Topology Spaces-II

Separated proximities, separation axioms satisfied by the closure operators induced by RI – (LO - , EF-) proximities. The Lattice structure of the class of RI – (LO-, EF-) proximities compatible with a suitable closure operator. (Basic) nearness, near families, contiguities, contigual families, closure operators, proximities and contiguities induced by a (basic) nearnesses, merotopic spaces. Nearness preserving maps. Separated nearnesses. The class of basic nearnesses compatible with a symmetric closure space ( a proximity space, a contiguity space) and their Lattice structure. (30L)

Clans, clusters and cluster generated (concrete) nearnesses. Nearnesses are not clan generated structures. Classification of basic nearness : Riesz (RI -) nearnesses, Lodato (LO -) nearnesses and Efremovič (EF -) nearnesses, their characterization and relationship between them. Nearness spaces, cluster generated nearness spaces, contigual nearness spaces and proximal nearness spaces and relation between them.

Correspondence between the principle (strict) T₁ extensions of a T₁ topological space X and the cluster generated compatible LO – nearnesses on X ; the correspondence between principal T₁ compactification of X and the compatible contigual LO – nearnesses on X ; the correspondence between the principal T₁ linkage compactifications of X and the compatible proximal LO – nearnesses on X . The correspondence between EF – nearnesses on a Tychonoff space X and the T₂ compactifications of X. (35L)
References:
2. W. J. Thron – Topological Structures (Halt Reinhurt and Winster, 1966)

H: Advanced Complex Analysis-II

Spaces of continuous functions, Ascoli-Arzela theorem, Spaces of Analytic functions, Hurwitz’s theorem, Riemann mapping theorem. (20L)

Meromorphic function, Mittag-Leffler’s theorem. (10L)

Elliptic function, weistrass’s elliptic function p(z), addition theorem for p(z), differential equation satisfied by p(z), the numbers e₁, e₂, e₃. (35L)

References:
1. J. B. Conway – Functions of One Complex Variable
2. L. V. Ahlfors – Complex Analysis
3. W. Rudin – Real and Complex Analysis
4. E. C. Titchmarsh – Theory of Functions
5. E. T. Copson – Function of a Complex Variable
6. R. P. Boas – Entire Functions
7. H. Cartan – Analytic Functions

Paper – MPT405

Term Paper

Marks: 50

Term paper MPT405 is related with the special papers of the pure stream offered by the department in each session and the topic of the term paper will also be decided by the department in each session. However the mark distribution is 30 marks for written submission, 15 marks for seminar presentation and 5 marks for viva-voce.
SEMESTER-III

APPLIED MATHEMATICS STREAM

Paper – MAG301

(Methods of Applied Mathematics -I)

Methods of Applied Mathematics -I

Total Lectures : 65 (Marks – 50)

Integral Transforms

Fourier Transform and its properties, Inversion formula of F.T.; Convolution Theorem; Parseval’s relation. Applications. Outline of Finite Fourier transform and its inversion formula. (10L)

Laplace’s Transform and its properties. Inversion by analytic method and by Bromwitch path. Lerch’s Theorem. Convolution Theorem; Applications. (10L)

Integral Equations

Linear Integral Equation, Reduction of differential equation to integral equation, Existence, Uniqueness and iterative solution of Fredholm and Volterra Integral equations; examples, Solution of Fredholm integral equation for degenerate kernel; Examples, Faltung type(closed cycle type) integral equation, Singular integral equation; Solution of Abel’s integral equation. (20L)

Generalised Functions

Generalised function; Elementary properties; Addition, Multiplication, Transformation of variables. Generalized function as the limit of a sequence of good functions, Differentiation of generalized function. Simple examples, Antiderivative, Regularisation of divergent integral : Simple example, Fourier Transform of generalized function, Examples, Convergence of a sequence of generalized functions; Examples, Laplace transform of generalized function. (12L)

Operator Equations on Hilbert Spaces

Inner product spaces, Hilbert spaces; orthonormality; closedness, and completeness of sets, Fourier expansion, Reisz Fischer theorem. (Proof not reqd.). Isometric isomorphism between a separable Hilbert space and $l_2$. Linear operators on Hilbert space, continuity, boundedness, adjointness, self-adjointness, invertibility, boundedness and unboundedness of inverse.

References:
2. I. N. Sneddon – Fourier Transforms (MacGraw-Hill)
3. R. V. Churchill – Operational Methods
4. Lusternik & Sobolev – Functional Analysis
5. Erwin Lareyizey – Introductory Functional Analysis with Applications
7. F. G. Tricomi – Integral Equation (Interscience Publishers)
8. WE. V. Lovit. – Linear Integral Equations (Dover Publishers)
9. F. John – Partial Differential Equations
10. Williams - Partial Differential Equations
11. Epstein - Partial Differential Equations
12. Chester - Partial Differential Equations
13. Arnold – Ordinary Differential Equations

Paper – MAG302

(Methods of Applied Mathematics –II, Theory of Electro Magnetic Fields)

Unit-1

Methods of Applied Mathematics -II

Total Lectures : 40 (Marks – 30)

Linear ordinary differential equations; generalized solution, fundamental solution, inverse of a differential operator. Two-point boundary value problem for a second-order linear O.D.E. Green’s functions and its bilinear expansion, particular integral, Analogy between linear simultaneous algebraic equations and Linear differential equation. (6L)

Mathematical models and initial boundary value problems of 2nd order partial differential equation (PDE); wellposedness; necessity of classification and canonical forms. Invariance of nature of an equation and its characteristics under coordinate transformation; transformation
of semilinear 2nd order PDE in two independent variables; linear transformations, and linear PDE’s with more than two independent variables. (5L)

Linear hyperbolic PDE’s in two independent variables Cauchy problem. Cauchy-Kowalasky theorem (statement only) reason for restriction on cauchy ground curve. Riemann-Green function. Domain of dependence and influence. Possible discontinuities of solutions; d’Alembert’s solution and meaning of generalized solution. (6L)

Linear parabolic equations: Heat equation in two independent variables, solution of Cauchy problem using Dirac Delta function and Fourier transform, maximum principle for initial – boundary value (for Dirichlet boundary condition) problem, uniqueness and stability of solution.

Methods of Eigen function expansion and Green’s function; Separation of variables, formulation of eigenvalue problems related to wave, heat and Laplace equations. (8L)

Linear elliptic equation: Laplace equation: boundary value problems of Dirichlet, Neumann and Robin. Greens formulas involving Laplacian; mean value theorem, maximum principle, uniqueness and stability of solutions; Dirichlet principle, Rayleigh-Ritz method. (5L)

Greens function for Dirichlet problem on Laplace eqn. its properties and methods of construction. Method of images. Method of conformal mapping for 2-dimentional problem with problem of a unit circle as an example. Bilinear expansion for Green’s function; Green’s function for heat equation by the method of Eigen function expansion and Bilinear expansion for Dirac Delta function. (10L)

References:

2. Stakgold – *Greens Functions and Boundary Value Problems* (John Wiley & Sons.)
4. V. S. Vladimov – *Equations of Mathematical Physics* (Marcel Danker, Inc. N.Y.)
5. Tikhnov & Samarski – *Equations of Mathematical Physics*

Unit-2

*Theory of Electro Magnetic Fields*

Total Lectures: 25 (Marks – 20)
Empirical basis of Maxwell’s Equations: Coulomb’s law, Gauss’ law, Electrostatic potential, Steady current, Equation of continuity of charge, Biot-Savart’s law, Magnetic induction, Ampere’s law, Faraday’s law, Maxwell’s equations for electromagnetic field and their empirical basis. Material equations, Conditions at an interface, Electromagnetic potentials, Electromagnetic energy, Poynting theorem. (15L)

Application of Maxwell’s Equations:
Plane electromagnetic Waves in vacuo, dielectric and conducting media, Group velocity and phase velocity, Retarded and accelerated potentials, Reflection and Refraction of plane waves at the plane boundary between two dielectrics, Field of a point charge in uniform motion. (10L)

References:

Paper – MAG303

(Continuum Mechanics-II, Dynamical Systems)

Unit-I
Continuum Mechanics-II

Total Lectures : 40 (Marks – 30)

deformations. Compatibility equations, Relative deformation gradient tensor, relative stretch
tensors and relative rotation tensor. Rate-of–strain tensor–its principal values and invariants
rate-of rotation tensor – vorticity vector; velocity gradient tensor, General principles of
momenta balance; Euler’s laws of motion. Body forces and contact forces. Cauchy’s laws of
motion: Stress equation of motion and symmetry of stress tensor for non-polar material.
Energy balance – first and second laws of thermodynamics. (20L)
Constitutive equation (stress-strain relations) for isotropic elastic solid. Elastic modulii.
Strain-energy function. Beltrami-Michel compatibility equations for stresses. Equations of
equilibrium and motion in terms of displacement. Fundamental boundary value problems of
elasticity and uniqueness of their solutions (statement only). Saint-Venant’s principle –
solution of simple problems. Wave propagation in an infinite elastic medium, Waves of
distortion and dilatation. (20L)

References :

1. Leigh, D. C. – Non-Linear Continuum Mechanics (MacGraw-Hill)
2. Truesdell, C – Continuum Mechanics
3. Chung, T. J. – Contumum Mechanics (Prentice-Hall)
5. Sokolnikoff, I. S. – Mathematical Theory of Elasticity
8. Schlichting, H. – Boundary Layer Theory
10. F. Chorlton – A Text Book of Fluid Mechanics
11. Kolin, Kebel & Roze – Theoretical Hydromechanics
13. J. Bansal – Viscous Flow Theory

Unit-2

Dynamical Systems

Total Lectures: 25          Marks: 20

Dynamical Systems : Phase variables and Phase space, continuous and discrete time
systems, flows(vector fields), maps (discrete dynamical systems), orbits, asymptotic states,
fixed (equilibrium) points periodic points, concepts of stability and SDIC (sensitive
dependence of initial conditions) chaotic behaviour, dynamical system as a group. (6L)
Linear systems: Fundamental theorem and its application. Properties of exponential of a matrix, generalized eigenvectors of a matrix, nilpotent matrix, stable, unstable and center subspaces, hyperbolicity, contracting and expanding behaviour. (6L)

Nonlinear Vector Fields: Stability characteristics of an equilibrium point. Liapunov and asymptotic stability. Source, sink, basin of attraction. Phase plane analysis of simple systems, homoclinic and heteroclinic orbits, hyperbolicity, statement of Hartmann-Grobman theorem and stable manifold theorem and their implications. (6L)

Liapunov function and Liapunov theorem. Periodic solutions, limit cycles and their stability concepts. Statement of Lienard’s theorem and its application to vander Pol equation, Poineare-Bendixsom theorem (statement and applications only), structural stability and bifurcation through examples of saddle-node, pitchfork and Hopf bifurcations. (7L)

References:

2. Strogartz – Non-linear Dynamics
5. Arnold – Ordinary Differential Equations

Paper – MAS304

(Special Paper-I)

Total Lectures: 65

Marks: 50

A: Viscous Flows, Boundary Layer Theory and Magneto Hydrodynamics-I

Viscous Flows

Some exact solutions of Navier – Stokes’ Equations: the flow due to suddenly accelerated plane wall; the flow near an oscillating plane wall; plane stagnation point flow (Hiemenz flow); the flow near a rotating disk; Hele-shaw flow; Bodewadt flow. (18L)
Navier-Stokes equations in non-dimensional form; Reynolds number. Creeping motion; hydrodynamical theory of lubrication; Stokes’s flow past a sphere and a cylinder: Stokes paradox; Oseen approximation, Oseen’s solution for a sphere. (18L)

**Laminar Boundary Layer Theory**

Concept of boundary layer: Prandtl’s assumptions. Two dimensional B.L. Equations for flow over a plane wall: Boundary layer on a flat plate; Blasius-Topfer solution, ‘Similar solutions’ of the B. L. equations: B. L. flow past a wedge; B. L. flow along the wall of a convergent channel; B. L. flow past a circular cylinder; (20L)

Separation of boundary layer.

The spread of a jet:

(i) plane free jet (two-dimensional jet),

(ii) circular jet (axisymmetric jet).

Prandtl-Mises transformation:

Karman momentum integral equation. Karman – Pohlhausen method: simple applications. (9L)

**References:**

5. J. A. Shercliff: A text Book of Magneto hydrodynamics

**B: Elasticity-I**

1. Generalised Hooke’s law Orthotropic and transversely isotropic media. Stress-strain relations in isotropic elastic solid. (5L)
Solution of torsion problem for simple sections Method of sol. of torsion problem by conformal mapping. (30L)


References:

1. Y. A. Amenzade – Theory of Elasticity (MIR Pub.)
4. W. Nowacki – Thermoelasticity (Addison Wesley)

C: Elasticity and Theoretical Seismology-I

Elasticity
Generalised Hooke’s law. Transversely isotropic media. Stress-strain relations in isotropic media. (10L)


Solution of torsion problem for simple sections. (20L)


Potential energy of deformation. Reciprocal theorem of Betti and Rayleigh. Theorem of minimum potential energy. (20L)


References:

1. Y. A. Amenzade – Theory of Elasticity (MIR Pub.)
4. *W. Nowacki* – Thermoelasticity (*Addison Wesley*)

**D: Applied Functional Analysis-I**


Multilinear forms. Analyticity Theorems. Non-linear Volterra operators. (15L)

**References:**

6. Dunford and Schwartz-*Linear operators*, vol. 1 & 11.
7. S. G. Krein-Linear Differential Equations in a Banach space.

**Paper – MAS305**

**(Special Paper-II)**

**Total Lectures : 65** (Marks – 50)

**A: Quantum Mechanics -I**

1. Transformation Theory : Adjoint operator, Hermitian operator, Projection operator, Degeneracy, Unitary transformation, Matrix representation of wave functions and operators, Change of basis, Transformation of matrix elements, Dirac’s Bra and Ket
notation, Completeness and normalization of eigen functions, Common set of eigen
functions of two operators, Compatibility of observables. (15L)

2. Symmetries and Invariance : Angular momentum eigenvalues and eigenfunctions,
Spin, Addition of two angular momenta, Rotation groups, Identical particles, Pauli
exclusion principle, Invariance and conservation theorems. (15L)

3. Relativistic Kinematics : Klein-Gordon equation, Dirac equation for a free particle
and its Lorentz covariance, Hole theory and positron, Electron spin and magnetic
moment. (15L)

4. Approximation Methods (time-independent)
Rayleigh-Schrödinger perturbation method, An harmonic oscillator, Stark effect in
hydrogen atom, Zeeman effect, Ground state energy of helium atom. (10L)

5. Elements of Second Quantization of A System : Creation and Annihilation operator,
Commutation and Anti-commutation rules, Relation with Statistics - Bosons and
Fermions. (10L)

References :

2. B. H. Bransden & C. J. Joachain – Introduction to Quantum Mechanics (Oxford
7. T. Y. Wu and T. Olmura – Quantum Theory of Scattering (Prentice Hall, New Jersey,
1962)
Press, Oxford, 1965)

B: Advanced Operations Research-I

Non-linear Optimization : Local and global minima and maxima, convex functions and their
properties, Method of Lagrange multiplier. (8L)
Optimality in absence of differentiability, Slater constraint qualification, Karlin’s constraint qualification, Kuhn-Tuckers Saddle point optimality conditions, Optimality criterion with differentiability and convexity, separation theorems, Kuhn-Tuckers sufficient optimality theorem. (10L)

Unconstrained Optimization: Search method: Fibonacci search, Golden Section search; Gradient Methods: Steepest descent Quasi-Newton’s method, Davidon-Fletcheher-Powell method, Conjugate direction method (Fletcheher-Reeves method). (15L)

Optimality conditions: Kuhn-Tucker conditions – non negative constraints (6L)

Quadratic Programming: Wolfe’s Modified Simplex method, Beale’s method (8L)

Separable convex programming, Separable Programming Algorithm. (6L)

Network Flow: Max-flow min-cut theorem, Generalized Max flow min-cut theorem, Linear Programming interpretation of Max-flow min-cut theorem, Minimum cost flows, Minflow max-cut theorem. (12L)

References:
17. Luenberger – *Introduction to Linear and Non-Linear Programming*
18. S. Dano – *Non-Linear and Dynamic Programming*
22. M. C. Joshi and K.M. Moudgalya, Optimization theory and Practice, Narosa Publishing House, New Delhi
24. M. A. Bhatti, Practical Optimization Methods, Springer -Verlag

C: Inviscid Compressible Flows and Turbulence-I
Basic thermodynamics; Equations of state; Polytropic gases. Euler’s equations of Motion; conservation of energy. Circulation theorem; Propagation of a small disturbance : Sound waves. Steady isentropic motion : Bernoulli’s eqn. Subsonic and supersonic flows. Irrotational flow : velocity potential; Bernoulli’s eqn. for unsteady flow. Stream function for steady two-dimensional motion. Steady flows through stream tubes, De Laval Nozzle. (30L) Method of characteristics, unsteady one-dimensional flow. Normal and oblique shock relations; shock polar diagram. (10L)


References:
5. J. A. Shercliff: A text Book of Magneto hydrodynamics

D: Computational Fluid Dynamics-I


References:
1. Peter Linz – Theoretical Numerical Analysis, An Introduction To Advance Technique (John Wiley & Sons.)

**SEMESTER-IV**

**APPLIED MATHEMATICS STREAM**

**Paper – MAG401**

(Continuum Mechanics-III)

Total Lectures : 65 (Marks – 50)

Stokes stream function. Vortex motion-vortex surface, vortex tube and vortex filament. Fundamental properties (Helmholtz properties) of vortex motion. Velocity field due to a distribution of vorticity. Velocity field due to a closed vortex filament. (40L)


**Linearly viscous incompressible fluid:**


**References :**

1. Leigh, D. C. – *Non-Linear Continuum Mechanics* (MacGraw-Hill)
2. Truesdell, C – *Continuum Mechanics*
5. Sokolnikoff, I. S. – *Mathematical Theory of Elasticity*
7. Pai, S. L. – *Viscous Flow Theory*
8. Schlichting, H. – *Boundary Layer Theory*
10. F. Chorlton – *A Text Book of Fluid Mechanics*
11. Kolin, Keibel & Roze – *Theoretical Hydromechanics*
13. J. Bansal – *Viscous Flow Theory*
15. Truesdell, C – *Continuum Mechanics*
18. Sokolnikoff, I. S. – *Mathematical Theory of Elasticity*
Unit-1

Elements of Quantum Mechanics

Total Lectures : 40 (Marks – 30)

1. **Origin of the Quantum Theory :**

2. **Basic Concepts :**
   Wave function of a free particle, Uncertainty and Complementarity principles, Gedanken experiments, wave packet, Schrödinger wave equation, Statistical interpretation of the wave function, Formal solution of the Schrödinger equation. (10L)

3. **Simple Applications (exact solutions) :**
   One dimensional potential step, Potential barrier, Square-well potential, Linear harmonic oscillator, Three-dimensional box potential, Spherically symmetric potential, Hydrogen atom bound-state problems. (10L)

4. **Dynamical Variables and Operators :**
   Operators corresponding to physical observables, Expectation values of observables, The virial theorem, Eigenfunction and eigenvalues of operators, Discrete and continuous spectra, Commutativity of operators, Heisenberg’s uncertainty relations, The minimum uncertainty product, Heisenberg’s equation of motion for operators. (5L)

**References :**

Unit-2

Chaos and Fractals

Total Lectures: 25 (Marks – 20)

Chaos and Fractals: Examples, graphical analysis, orbits, phase diagrams fixed and periodic points stable and unstable sets smooth maps and conditions for stable and unstable periodic points hyperbolicity. (10)

SDIC, topological transivity (mixing) and Devaney’s definition of chaos, binary decimal representation of numbers and saw tooth map. One parameter family of maps and bifurcations (through examples only) topological conjugacy, Logistic map, period doubling route to chaos (10L)

Cantor sets, examples of fractals, definitions of topological and capacity dimensions, Horse shoe and the theorem: “period 3 implies chaos”. (5L)

References:

2. Strogatz – Non-linear Dynamics
3. R. L. Devaney – A First Course In Chaotic Dynamical Systems
5. R. L. Devaney – An Introduction To Chaotic Dynamical System (Addison-Wesley 1987)

Paper – MAS403

(Special Paper-III)

Total Lectures: 65 Marks: 50

A: Viscous Flows, Boundary Layer Theory & Magneto-Hydrodynamics-II
Electromagnetic equations for moving media, Ohm’s law including Hall current, Lorentz force. MHD approximations. Stress-tensor formulation of Lorentz force; frozen-in-magnetic field. Alfven’s Theorem; Alfven waves. Equations of motion and induction; their nondimensional forms; dimensionless parameters, Lundquist’s criterion. Energy equation: Viscous and Joule dissipation, Poynting theorem. Boundary conditions. (25L)

Steady viscous incompressible flows: unidirectional flow under a transverse magnetic field: decoupling of MHD equations. Hartmann flow; Couette flow. Flow through a rectangular duct. Unsteady incompressible flows. Rayleigh’s problem. MHD waves: propagation of small disturbances; plane waves; Reflection and transmission of plane harmonic waves; existence of finite amplitude MHD waves. Alfven waves with ohmic damping; Skin effect. (25L)

Magnetohydrostatics: equilibrium-configurations, Pinch effect, force-free fields, nonexistence of force-free field of finite extent. General solution for a force-free field, special cases.

Dynamo problem, Cowling’s theorem, Ferraro’s law of isorotation. (15L)

References:

5. J. A. Shercliff: A text Book of Magnetohydrodynamics

B: Elasticity-II

Vibration problems: Longitudinal vibration of thin rods, Torsional vibration of a solid circular cylinder and a solid sphere. Free Rayleigh and Love waves. (15L)


Magneto-elasticity: Interaction between mechanical and magnetic field. Basic equations Linearisation of the equations. (20L)
References:
1. Y. A. Amenzade – Theory of Elasticity (MIR Pub.)
4. W. Nowacki – Thermoelasticity (Addison Wesley)

C: Elasticity and Theoretical Seismology-II

Theoretical Seismology

Theory of elastic waves; Motion of a surface of discontinuity – kinematical condition and dynamical conditions. Kirchoff’s solution of inhomogeneous wave equation. (20L)
Reflection and refraction of elastic body waves. (10L)
Surface waves: Rayleigh, Love and Stonely waves (10L)
Dispersion and Group Velocity of elastic body waves. (10L)
Some problems: Application of pressure and twist on the walls of a spherical cavity in an elastic medium. (10L)

Line source and point source on the surface of a semi-infinite elastic medium. (5L)

References:
1. Y. A. Amenzade – Theory of Elasticity (MIR Pub.)
4. W. Nowacki – Thermoelasticity (Addison Wesley)
9. B.L. N. Kennett, Seismic wave propagation in Stratified Media, CUP
10. K.E. Bullen, An Introduction to the theory of Seismology, CUP.
D: Applied Functional Analysis-II


References:


Paper – MAS404
(Special Paper-IV)

Total Lectures : 65 (Marks – 50)

A: Quantum Mechanics -II

Collision Theory : Basic concepts, Cross sections, Laboratory and center-of-mass coordinates, Rutherford scattering. Quantum mechanical formulation — time independent and time-dependent, Scattering of a particle by a short-range potential, Scattering by Coulomb potential, Scattering by screened Coulomb field, Scattering by complex potential. (15L)
Integral Equation Formulation: Lippmann-Schwinger integral equation and its formal solutions, Integral representation of the scattering amplitude, Convergence of the Born Series, Validity of Born approximation, Transition probabilities and cross sections. (12L)
Semi-Classical Approximations : WKB approximation, Eikonal approximation. (8L)
Variational Principles in the Theory of Collisions: General formulation of the variational principle, Hulthen, Kohn-Hulthen and Schwinger variational methods, Determination of Phase shifts, Scattering length and scattering amplitude for central force problems, Bound (minimum) principles. (20L)

Analytic Properties of Scattering Amplitude: Jost function, Scattering matrix, Bound states and resonances, Levinson theorem, Dispersion relations, Effective range theory. (10L)

References:


B: Advanced Operations Research-II

Dynamic Programming: Characteristics of Dynamic Programming problems, Bellman’s principle of optimality (Mathematical formulation)
Model – 1: Single additive constraint, multiplicatively separable return,
Model – 2: Single additive constraint, additively separable return,
Model – 3: Single a multiplicative constraint, additively separable return,
Multistage decision process – Forward and Backward recursive approach, Dynamic Programming approach for solving linear and non-linear programming problems, Application – Single-item N-period deterministic inventory model. (25L)
Geometric Programming: Elementary properties of Geometric Programming and its applications. (8L)

Queuing Theory: Introduction, characteristic of Queuing systems, operating characteristics of Queuing system. Probability distribution in Queuing systems. Classification of Queuing models. Poisson and non-Poisson queuing models (32L)

References:
7 S. Dano – *Non-Linear and Dynamic Programming*

C:  *Inviscid Compressible Flows and Turbulence-II*

Turbulence:

Spread of turbulence: Mixing zone between two parallel flows, (two-dimensional) turbulent wake behind (i) symmetrical cylinder (ii) a row of parallel rods. Turbulent flow through smooth circular pipes; Seventh power velocity distribution law; turbulent boundary layer on a flat plate. (20L)

Statistical approach; Introductory concepts; double correlation between velocity components, longitudinal and lateral; correlations in homogeneous turbulence; Eulerian correlation with respect to time, Taylor’s one-dimensional energy spectrum. Energy relations in turbulent flows. (15L)

References:


References :
1. Peter Linz – *Theoretical Numerical Analysis, An Introduction To Advance Technique* (John Wiley & Sons.)

**Paper – MAT405**

Term Paper
Term paper MAT405 is related with the special papers of the applied stream offered by the department in each session and the topic of the term paper will also be decided by the department in each session. However the mark distribution is 30 marks for written submission, 15 marks for seminar presentation and 5 marks for viva-voce.