SYLLABUS FOR PH.D. COURSE WORK AND M.PHIL. IN MATHEMATICS

Effective from 2020-2021

The University of Burdwan
Burdwan-713104
West Bengal

Preamble

The M.Phil. in Mathematics is a four-semester programme (two years) with 42 credits. 12, 12, 8, 6 and 4 credits are given for dissertation, 3Elective courses, 2 term papers, 2 Compulsory courses and viva-voce respectively. The Ph.D. Course Work is a programme of one-semester which coincides with the 1st Semester of M.Phil. in Mathematics.

Objectives

- Impart teaching so that the students could develop critical thinking ability about the fundamental aspects of mathematics.
- Imparting knowledge in research work in various emerging fields of mathematics and its applications.
- Train the students with mathematical knowledge and computation techniques for carrying out scientific investigations independently.

Pre-requisite

The students should possess the knowledge on the courses taught in the M.A./M.Sc. in Mathematics.

Programme Outcomes

- Knowledge in the topics required for undertaking specialized research in various fields of Mathematical Sciences.
- Identification of unsolved relevant problem in a specific field.
- Articulating ideas and strategies for addressing a research problem.
- Undertaking original research on a particular topic.
- Effectively communicating research, through journal publications and conference presentations to the mathematics community.
- Disseminating research to a broader audience.
- Understand the historical and contemporary role of mathematics and be able to place the discipline properly in the context of other human intellectual achievement.

Program Specific Outcomes

- Generation of publications in reputed journals.
- Scopes for interaction with international researchers and developing collaborations.
- Demonstrate the highest standard of ethics in research.
- Opportunities to research students for communication (and discussion) of advanced mathematical topics to post graduate students.
- Produce next generation researchers in mathematics.
- Investigate and solve unfamiliar problems.
- Investigate and apply mathematical problems and solutions in a variety of contexts related to science, technology, business and industry, and illustrate these solutions using symbolic, numeric, or graphical methods.
- Communicate mathematical ideas in both oral and written forms.
- Gain experience investigating the real world problems and learn to how to apply mathematical ideas and models to those problems.

PROFILE

Semester I

Course				Lect. Hr /week	Dur. of Exam (in H)	Marks	Credit
Course code	Type	T/P	Name				
PHSMA101/MPSMA101	COM	T	Research Methodology	4	2	50	4
PHSMA102/ MPSMA102	COM	T	Research and Publication Ethics	2	1	25	2
PHSMA103/ MPSMA103	EL	T	**Vide Appendix I**	4	2	50	4
PHSMA104/ MPSMA104	TM	T	N.A.	N.A.	N.A.	50	4

Abbreviation used: T/P → Theory/Practical; COM→ Compulsory, EL→ Elective, TM→ Term paper

Appendix I

Basket of courses for Elective (Only one course has to be chosen from the basket)

PHSMA103-1/ MPSMA103-1: Mechanics in Fluid media
PHSMA103-2/ MPSMA103-2: Advanced Operations Research & Optimization
PHSMA103-3/ MPSMA103-3: Special Relativity & Quantum Mechanics
PHSMA103-4/ MPSMA103-4: Differential Geometry of Manifolds
PHSMA103-5/ MPSMA103-5: Algebra
PHSMA103-6/ MPSMA103-6: Advanced Analysis
PHSMA103-7/ MPSMA103-7: Real and Complex Analysis

Semester II

Course				Lect. Hr /week	Dur. of Exam (in H)	Marks	Credit
Course code	Type	T/P	Name				
MPSMA201	EL	T	**Vide Appendix II**	4	2	50	4
MPSMA202	EL	T	**Vide Appendix III**	4	2	50	4
MPSMA203	TM	T	N.A.	N.A.	N.A.	50	4

Abbreviation used: T/P → Theory/Practical, EL → Elective, TM→ Term paper

Appendix II

Basket of courses for Elective (Only one course has to be chosen from the basket)

MPSMA201-1: Advanced Analysis-II
MPSMA201-2: Measure and Integration
MPSMA201-3: Advanced Operations Research & Optimization-II
MPSMA201-4: Boundary Layer Theory and Turbulence
MPSMA201-5: General Relativity & Relativistic Quantum Mechanics

Appendix III

Basket of courses for Elective (Only one course has to be chosen from the basket)

MPSMA202-1: Riemannian manifolds and contact manifolds
MPSMA202-2: Advanced Complex Analysis
MPSMA202-3: Algebra-II

MPSMA202-4: Computational Optimization
MPSMA202-5: Chaos Theory and Fractals
MPSMA202-6: Quantum Theory of Scattering & Quantum Field Theory

Semester III & IV

Course		Marks	Credit
Course code	Type		
MPSMA301	Dissertation	150	12
MPSMA302	Viva-voce	50	4

PHSMA101/MPSMA101: Research Methodology

Marks: 50 **Credits: 4** **Total Lectures: 50**

Group – A (Research Methodology) (25 Marks)

Introduction to Research Methodology: Meaning of Research, Objectives of Research, Motivations in Research, Ethics of research (Legal issues, copyright, plagiarism), Types of Research, Research Approaches, Significance of Research, Research Methods v/s Methodology, Research and Scientific Methods, Research Process, Criteria of good Research. [8]

Defining the Research Problem: What is Research Problem?, Selecting the problem, Necessity of and Techniques in defining the problem. [3]

Review of Literature: Purpose of the Review, Identification of the related Literature, Organizing the related Literature, Archive. [4]

Research Report: General format of the Research report, writing technical research report, Writing a research proposal, research paper, chapter writing, Ph.D. thesis, Erratum, Proof reading, Keywords and Phrases, Mathematical subject Classifications and indexing, Short communication, fast track communication of a research paper, Poster/Oral presentation, Plenary talks, Invited talks of a conference/ workshop. [10]

Recommended Books:

1. C. R. Kothari, Research Methodology: Methods and Techniques, New Age International (P) Ltd., New Delhi, 2010.
2. Lokesh Koul, Methodology of Educational Research, Vikas Publishing House, 1984.
3. Ranjit Kumar, Research Methodology: A Step-by-Step Guide for Beginners, SAGE Publications, 3rd Edition, 2011.

Group –B (Computer Fundamentals) (25 Marks)

Introduction to computer, Basic concept of computer hardware, software, operating systems, Algorithm and Flowchart, programming languages, representation of numbers in computers; Scientific text processing packages; Statistical packages; Curves, surface drawing packages; Basic concepts of programming with Mathematica and MATLAB. [25]

Recommended Books:

1. G. C. Layek, A. Samad and S. Pramanik; Computer Fundamentals, Fortran 77, C and Numerical Programs, Levant Books (Sarat Book House).

PHSMA102/MPSMA102: Research and Publication Ethics (RPE) - Course for awareness about the publication ethics and publication misconducts

Marks: 25 **Credits: 2** **Total Lectures: 30**

RPE 01: Philosophy and Ethics(3 hrs.)

1. Introduction to philosophy: definition, nature and scope, concept, branches
2. Ethics: definition, moral philosophy, nature of moral judgements and reactions

RPE 02: SCIENTIFICCONDUCT (5hrs.)

1. Ethics with respect to science and research
2. Intellectual honesty and research integrity
3. Scientific misconducts: Falsification, Fabrication, and Plagiarism (FFP)
4. Redundant publications: duplicate and overlapping publications, salami slicing
5. Selective reporting and misrepresentation of data

RPE 03: PUBLICATION ETHICS (7 hrs.)

1. Publication ethics: definition, introduction and importance
2. Best practices / standards setting initiatives and guidelines: COPE, WAME, etc.
3. Conflicts of interest
4. Publication misconduct: definition, concept, problems that lead to unethical behavior and vice versa, types
5. Violation of publication ethics, authorship and contributorship
6. Identification of publication misconduct, complaints and appeals
7. Predatory publishers and journals PRACTICE

RPE 04: OPEN ACCESS PUBLISHING (4 hrs.)

1. Open access publications and initiatives
2. SHERPA/ROMEO online resource to check publisher copyright & self-archiving policies
3. Software tool to identify predatory publications developed by SPPU
4. Journal finder / journal suggestion tools viz. JANE, Elsevier Journal Finder, Springer Journal Suggester, etc.

RPE 05: PUBLICATION MISCONDUCT (4hrs.)

A. Group Discussions (2 hrs.)

1. Subject specific ethical issues, FFP, authorship
2. Conflicts of interest
3. Complaints and appeals: examples and fraud from India and abroad

B. Software tools (2 hrs.)

Use of plagiarism software like Turnitin, Urkund and other open source software tools

RPE 06: DATABASES AND RESEARCH METRICS (7hrs.)

A. Databases (4 hrs.)

1. Indexing databases
2. Citation databases: Web of Science, Scopus, etc.

B. Research Metrics (3 hrs.)

1. Impact Factor of journal as per Journal Citation Report, SNIP, SJR, IPP, CiteScore

2. Metrics: h-index, g index, i10 index, altmetrics

References

Bird, A. (2006). Philosophy of Science. Routledge.

MacIntyre, Alasdair (1967) A Short History of Ethics. London.

P. Chaddah, (2018) Ethics in Competitive Research: Do not get scooped; do not get plagiarized, ISBN:978 9387480865

National Academy of Sciences, National Academy of Engineering and Institute of Medicine. (2009). On Being a Scientist: A Guide to Responsible Conduct in Research: Third Edition. National Academies Press.

Resnik, D. B. (2011). What is ethics in research & why is it important. National Institute of Environmental Health Sciences, 1-10. Retrieved from https://www.niehs.nih.gov/research/resources/bioethics/whatis/index.cfm Beall, J. (2012). Predatory publishers are corrupting open access. Nature, 489(7415), 179–179. https://doi.org/10.1038/489179a

Indian National Science Academy (INSA), Ethics in Science Education, Research and Governance(2019), ISBN:978-81-939482-1-7. http://www.insaindia.res.in/pdf/Ethics Book.pdf

PHSMA103/MP103: Elective Courses

PHSMA103-1/MPSMA103-1:Mechanics in Fluid media

Marks: 50 **Credits: 04** **Total lectures Hours: 50**

Continuum mechanics: basic laws, continuum hypothesis for fluids, role of fluid viscosity, different non-Newtonian fluid models and applications, Stokes' hypothesis on stress and strain relations, homogeneous and isotropic fluid medium, Navier-Stokes equations, Significance of Reynolds number on flows, some exact solutions of Navier-Stokes equations, approximate equation for slow motion, Stokes solution and paradox, its modifications by Oseen, high Reynolds number flows. [35]

Governing equations for electrically conducting fluid, Maxwell equations, Lorentz force, MHD approximations, magnetic induction equation. [15]

Recommended Books:

1. G. K. Batchelor, *An Introduction to Fluid Dynamics*, Cambridge University Press, 2005.
2. S.W. Yuan, *Foundations of Fluid Mechanics*, Prentice – Hall International, 1970.
3. H. Schlichting, *Boundary Layer Theory*, Springer, 2003
4. P.K. Kundu and I.M. Cohen, *Fluid Mechanics*, 3rd Ed., Academic Press, (2004).
5. F.S. Sherman, *Viscous Flow*, McGraw Hill International, (1990)
6. K. Muralidhar and G. Biswas, *Advanced Engineering Fluid Mechanics*, 2nd Ed., Alpha Science, (2005)
7. F.M. White, *Viscous Fluid Flow*, McGraw Hill International, (1991)

PHSMA103-2/MPSMA103-2: Advanced Operations Research & Optimization

Marks: 50 **Credits: 04** **Total lectures Hours: 50**

Inventory Control : Dynamic & probabilistic models. [15]
Interval Mathematics and Interval ranking. [10]
Soft computing methods for optimization. [15]
Optimization by MATHEMATICA, MATLAB and LINGO. [10]

Recommended Books:

1. Hadley, G. and Whitin, T.M., Analysis of inventory system, Prentice Hall, Englewood Cliffs, NJ, 1963
2. Arrow, K.J., Karlin, S. and Scraf, H., Studies in the mathematical theory of inventory and production, Stanford, calif, Standford University Press, 1958
3. S. S. Rao, Optimization-Theory and Applications, Wiley Eastern Ltd., 1977.
4. C. Mohan and K. Deep, Optimization Techniques, New Age Publishing, 2009.
5. Michalawich, Z. (1996), Genetic algorithms + Data structures = evolution Programs, Springer Verlag, Berlin, Third Edition, 1996.
6. Andrea E. Olsson, Particle Swarm Optimization: Theory, Techniques and Applications, Nova Science Publishers, 2011.
7. M. Aokie, *Introduction to Optimization Techniques: Fundamentals and Applications of Nonlinear Programming*, The Macmillan Company, 1971.
8. S. Wolfram, *The Mathematica Book*, Wolfram Media, 2003.

PHSMA103-3/MPSMA103-3: Special Relativity & Quantum Mechanics

Marks: 50 **Credits: 04** **Total lectures Hours: 50**

Group - A (Special Relativity) (25 Marks)

Empirical evidence for the constancy of c, frames of reference. [2]
Lorentz transformations. [4]
relativity of simultaneity; twin and other paradoxes. [4]
transformation laws for velocity, momentum, energy. [5]
mass-energy equivalence; force equations, kinematics of decays and collisions. [5]
Maxwell's equations in covariant form. [5]

Recommended Books:

1. Robert Resnick,Introduction to Special Relativity, Wiley.
2. Anadijiban Das,The Special Theory of Relativity: A Mathematical Approach,Springer-Verlag.
3. Wolfgang Rindler,Introduction to Special Relativity, 2nd ed,Oxford University Press.
4. Edwin F. Taylor and John Archibald Wheeler,Spacetime Physics: Introduction to Special Relativity, W. H. Freeman & Company, 1992.

Group –B (Quantum Mechanics)(25 Marks)

Postulates of quantum mechanics. [4]
Time-evolution of state vectors. [4]
Time-independent Schrodinger equation. [4]
Theory of harmonic oscillator. [4]
Angular momentum, Spin, Hydrogen atom. [4]
Approximate methods for bound states. [5]

Recommended Books:

1. B. H. Bransden and C. J. Joachain, Physics of atoms and molecules, Second Edition, *Pearson Education*, New Delhi, (2008).
2. D. Griffiths, Introduction to Quantum Mechanics, Second Edition, Pearson Education, New Delhi, (2014).

PHSMA103-4/MPSMA103-4: Differential Geometry of Manifolds

Marks: 50 **Credits: 4** **Total Lectures Hours: 50**

Differentiable manifolds: Definition and Examples of manifolds, Diffeomorphisms on manifolds, Tangent and Cotangent spaces, Tangential maps, Vector Fields on smooth manifolds, Lie Brackets and its properties, Integral Curves and Flows, 1-parameter group of transformations, Exterior algebra and Exterior derivatives. [20]

Riemannian manifolds: Affine connections, Riemannian and semi-Riemannian metrics, Riemannian connection, Riemann curvature tensor, Ricci tensor, Scalar curvature, Sectional Curvature, Semi-symmetric and quarter symmetric metric connections on Riemannian manifolds, Einstein manifolds and its generalizations, Manifolds of constant curvature and its generalizations, Some transformations (eg. Conformal transformation, Projective transformation, Concircular transformation, Conharmonic transformation) on Riemannian manifolds, Locally symmetric Riemannian manifolds due to Cartan and its generalizations, Product manifolds, Warped product manifolds. [20]

Submanifolds: Embedded Submanifolds, Immersed Submanifolds, Hypersurfaces of Riemannian manifolds, Induced connection and second fundamental form, Gauss and Weingarten formulae, Equations of Gauss, Codazzi and Ricci, Mean curvature, Totally geodesic and totally umbilical submanifolds, Minimal submanifolds. [10]

Recommended Books:

1. John M. Lee, *Introduction to Smooth Manifolds*, 2nd Ed., Springer-Verlag, 2012.
2. U. C. De and A. A. Shaikh, *Differential Geometry of Manifolds*, Narosa Publ. Pvt. Ltd, New Delhi, 2007.
3. William H. Boothby, *An Introduction to Differentiable Manifolds and Riemannian Geometry*, Academic Press, New York, 1975.
4. S. Lang, *Introduction to Differential Manifolds*, John Wiley and Sons, New York, 1962.
5. Manfredo P. Do Carmo, *Riemannian Geometry*, Birkhauser, Boston, 1992**.**
6. P. Petersen, Riemannian Geometry, Springer Verlag, 2006.
7. T. J. Willmore, *Riemannian Geometry*, Oxford University Press, 1997.
8. K. Yano and M. Kon, *Structure on Manifold*, World Scientific Publication, Singapore, 1984.

9. J. M. Lee, *Riemannian Manifolds*, An Introduction to Curvature, Springer-Verlag, 2005.
10. B. Y. Chen, Geometry of Submanifolds, Marcel Dekker. Inc., New York, 1973.

PHSMA103-5/MPSMA103-5: Algebra

Marks: 50 **Credits: 4** **Total Lectures Hours: 50**

(By a ring we always assume commutative ring with 1.)
Ideals, Quotient rings, nilpotent elements, Prime and Maximal ideals, Prime ideals , m-system, semi prime ideals, prime radical, Prime rings, nilradicals and Jacobson radical - their properties. DCC, operations on ideal, extension and contraction. [10]

Sub direct sum of rings: Definition, Fundamental properties, subdirectly irreducible rings, Boolean rings.
Integral Dependence and Valuation: Integral Theorem, The going-up theorem, Integrally closed integral domain, The going-down theorem, valuation ring. [15]

Primary Decomposition, Primary Decomposition in Noetherian rings, Artin rings
Discrete Valuation rings, Dedekind domains, fractional ideals. [10]

Real closed field, Algebraic and Transcendental subset, transcendence degree, order structure of hyper real field, η_1-set, isomorphism of η_1-field, Dedekind completion of η_1-sets. [15]

Recommended Books:

1. M. F Atiyah and I.G Macdonald: An Introduction to Commutative Algebra, Addision-Wesley Publishing company.
2. N.H McCoy: Theory of Rings, Macmillan, 1964
3. L. Gillman and M Jerrison: Rings of Continuous Functions, Springer, New York
4. D. S Dummit and R. M . Foote: Abstract Algebra Wiley; Third edition , 2011

PHSMA103-6/MPSMA103-6: Advanced Analysis

Marks: 50 **Credits: 04** **Total lectures Hours: 50**

Group- A (Topology)(25 Marks)

Axiomatic set theory, ordinal, ordinal and cardinal numbers and their properties. [5]
Normal spaces, Urysohn's lemma, Tietz's extension theorem, product spaces and their properties, quotient spaces, Uniform spaces. [5]
Compactification-One point compactification, Stone Cech compactification, Wallman compactification, Banachwisky compactification, Paracompact spaces and their properties. [10]
Proximity spaces, induced topology, proximal neighbourhood, quasi proximity spaces. [5]

Recommended Books:

1. James Dugundji: Topology, McGraw-Hill Inc.,US , 1988

2. R. Engelking: General Topology, Heldermann Verlag Berlin
3. R. E Chandler: Hausdorff Compactification, Marcel Dekker, New York, 1976
4. J. Nagata: Modern Dimension Theory, North-Holland Publishing Company, Amsterdam
5. Russel C. Walker: The Stone-Cech Compactification, Springer
6. J.L Kelley: General Topology, D. Van Nostrand Company Inc., 1955
7. J. R. Mukres: Topology, Pearson Education India; 2 edition ,2015

Group – B (Functional Analysis) (25 Marks)

Continuity of mappings: Weak continuity, demi continuity, hemi continuity, strongly hemi continuity, statistical continuity, I and I*-continuity. [7]

Hausdorff metric on 2^X, set valued maps. [2]

Metric fixed point theory of contraction mapping, contractive mapping and non expansive mappings and its applications, common fixed point theory in metric spaces. [8]

Summability methods: Asymptotic density, statistical convergence, I and I*-convergence of sequences in normed linear spaces, weak statistical and I-convergence of sequences of functions. [8]

Recommended Books:

1. Kazimierz Goebel and W.A. Kirk- Topics in Metric Fixed Point Theory, Cambridge University Press, 1990
2. Andrzej Granas and James Dugundji-Fixed Point Theory, Springer Verlag, 2003
3. M.C. Joshi and R.K. Bose-Some Topics in Non Linear Functional Analysis, Wiley Esatern Ltd., 1985
4. J.P. Aubin-Applied Abstract Analysis , John Wiley and Sons, N.Y., 1977
5. B.V. Limaye-Functional Analysis, New Age International (P) Ltd., Publishing,1996
6. E.Zeidler-Applied Functional Analysis, Springer Verlag, 1995
7. N. Dunford and J.T. Schwartz-Linear operators , Part-I General Theory (Vol.1) , Wiley, 1958
8. A.L. Brown and A.Page-Elements of Functional Analysis, Von Nostrand Reinhold Co., 1970
9. P.R. Halmos-A Hilbert Space Problem Book, Von Nostrand Prinuton, 1967
10. C.L. Devito-Functional Analysis and Linear Operator Theory, Addison Wesley, 1990
11. E. Kreyszig, Introductory Functional Analysis with Applications, John Wiley & Sons, 1978.
12. H. Dutta and B.E. Rhoades, Current Topics in Summability Theory and Applications, Springer Singapore, 2016

PHSMA103-7/MPSMA103-7: Real and Complex Analysis

Marks: 50 **Credits: 04** **Total lectures Hours: 50**

Group- A (Real Analysis) (25 Marks)

Differentiation of real functions. Dini derivatives and their properties. Monotonicity theorem. Example of a continuous nowhere differentiable function. Vitali's covering theorem in one dimension. Differentiability of monotone functions. Absolutely continuous functions and singular functions. [15]

Density of arbitrary linear sets. Lebesgue density theorem. [5]

The Perron integral: Definitions and basic properties, Comparison with Lebesgue integral and Newton integral. [5]

Recommended Books:

1. Hewitt and Stomberg, Real and Abstract Analysis, Springer-Verlag New York, 1965.
2. I. P. Natanson – *Theory of Functions of a Real Variable,* Vol. I & II, Fedrick Unger Publi. Co., 1961.
2. H. L. Royden – Real Analysis, PHI, 1989
3. S. Saks, Theory of the Integral, Dover Publications Inc.; 2nd, 2005.
4. W. Rudin, Real and Abstract Analysis, Springer, 1975.
5. M. E. Munroe, Introduction to Measure and Integration, Addison-Wesley, 1953.
6. E. W. Hobson, Theory of Functions of a Real Variable and the Theory of Fourier's Series, Vol I & II, Cambridge University Press, 1957.

Group- B (Complex Analysis) (25 Marks)

Fundamental Concepts: Analytic functions, Cauchy Riemann Equations, Cauchy's Theorems, Cauchy's integral formula, Applications of Cauchy's integral formula, Liouvilles theorems, Taylor's theorem, Laurent theorem, Cauchy Residue theorem, Contour integral, Zeros of holomorphic functions, Rouche's theorem, Counting of zeros and poles. [10]

Applications of Maximum modules principle and Jensen's formula: Jensen's formula, Picard Borel Theorem, Borel Caratheodary theorem, Hadamard three circle theorem, effect of small derivatives, Nevanlinna Theory. [15]

Recommended Books:

1. Serge Lang, Complex Analysis, IVth edition, Springer International Edition.
2. J. B. Conway**,** Functions of One Complex Variable.
3. R. E. Greene and S. G. Krantz, Function theory of one complex variables, American Mathematical Society.
4. S. G. Krantz, Complex Variables, Chapman & Hall.
5. L. V. Ahlfors, Complex Analysis, McGraw Hill.
6. A.S.B. Holland, Introduction to the theory of Entire Functions, Academic Press, New York, 1973.
7. Lee A. Rubel, Entire and meromorphic functions, Springer, 2001.

PHSMA104/MPSMA104: Term paper

Marks: 50 **Credits: 4**

[Term paper must comprise of either the report on new research works or review of research works on a current research topic done by the candidate under his/her capacity(solely or partially).]

MPSMA201: Elective Courses

MPSMA201-1: Advanced Analysis- II

Marks: 50 **Credits: 4** **Total Lectures Hours: 50**

Group- A (Topology –II) (25 Marks)

Topological ordered spaces, closed order and convex topological spaces, normally ordered spaces, compact ordered spaces, completely regular ordered spaces, uniform ordered spaces and its structure. [10]

Dimension theory-an introduction. [3]

Boolean algebra, Stone representation theorem, completion of Boolean algebra, separability in Boolean algebra. 10]

Density Topology: [2]

Recommended Books:

1. James Dugundji, *Topology*, McGraw-Hill Inc.,US , 1988.
2. R. Engelking, *General Topology*, Heldermann Verlag Berlin, 1989.
3. R. E Chandler, *Hausdorff Compactification*, Marcel Dekker, New York, 1976.
4. J. Nagata, *Modern Dimension Theory*, North-Holland Publishing Company, Amsterdam, 1965.
5. Russel C. Walker, *The Stone-Cech Compactification*, Springer-Verlag, Berlin Heidelberg, 1974.
6. J.L Kelley, *General Topology*, D. Van Nostrand Company Inc., 1955.
7. J. R. Mukres, *Topology*, Pearson Education India; 2 edition ,2015.

Group – B (Functional Analysis –II) (25 Marks)

Approximation theory in normed spaces, B* Algebra. [7]

Spectral Theory of linear operators in normed linear spaces: spectral properties compact operators, unbounded operators and its applications, numerical range of bounded linear operators. [13]

Geometric structure on normed linear spaces : Strict convexity, uniform convexity. [5]

Recommended Books:

1. B.V. Limaye, *Functional Analysis*, New Age International Publisher, 2017.
2. I.Singer, *Best approximationsin normed linear spaces by elements of linear subspaces* Springer Verlag, 1970.
3. Edgar R.Lorch, *Spectral Theory*, Dover, 2018.
4. G.Bachman and L.N.Narici, *Functional Analysis* , Dover, 2000.
5. E. Kreyszig, *Introductory Functional Analysis with Applications*, John Wiley & Sons, 1978.
6. W.Rudin, *Functional Analysis*, Tata Mc-Graw Hill, 1986.

MPSMA201-2: Measure and Integration

Marks: 50 **Credits: 04** **Total lectures Hours: 50**

Measures: Rings and σ-Rings, Lemma on Monotone Classes, Measure as a set function, Outer measures, Extension of measures, Measurable Covers, Completion of a Measure, Measurable Spaces, Measurable Functions, Limits of Measurable Functions, Simple Functions, Measure Spaces, Convergence in Measure, Almost Uniform Convergence, Egoroff's Theorem, Integrable Simple Functions, Integrable functions, Mean Convergence. [25]

Integration over Locally Compact Topological Spaces: Continuous Functions with Compact Support, G_∂-sets , F_σ-sets, Baire-sets , Borel sets, Regularity of Baire and Borel Measures, Content, Regular Content, Regular Borel Extention of a Baire Measure, Integration of Continuous Functions with Compact Support, Approximation of Baire Functions, Approximation of Borel Function, Riesz-Markoff Representation Theorem. [25]

Recommended Books:

1. Sterling K.Berberian, *Measure and Integration*, The Macmillan Company, New York, 1967.
2. P.R.Halmos, *Measure Theory*, Van Nostrand, Princeton, 1950.

MPSMA201-3: Advanced Operations Research & Optimization - II

Marks: 50 **Credits: 04** **Total lectures Hours: 50**

Simulation: Introduction, Types of simulation, Steps of simulation process, Advantages and Disadvantages of simulation, Monte Carlo Simulation, Random number generation, Simulation of Inventory, Queueing, investment and PERT problems. [15]

Network Flow: Introduction, Minimum path problem, Potential difference problem, Network flow problem and graphical representation, Max flow min cut theorem, Generalised maximum flow problem. [10]

Advanced Game Theory: Cooperative and non-cooperative games, Infinite Antagonistic game, Continuous game, Separable game, bi-matrix game. [10]

Information Theory: Introduction, Measure of information, Entropy (expected information), Marginal, joint and conditional entropies, Mutual information, Information process by a channel, Shannon Fano encoding procedure. [15]

Recommended Books:

1. J. K. Sharma, *Operations Research: Theory and Applications*, Macmillan, 1997
2. S. D. Sharma, H. Sharma, *Operations Research: Theory, Methods and Applications*, KedarNath Ram Nath, 1972
3. K. Swarup, P. K. Gupta, M. Mohan, *Operations Research*, Sultan Chand & Sons, 1978
4. F. S. Hillier, G. J. Lieberman, Introduction to Operations Research, McGraw-Hill, 2001
5. A. K. Bhunia and L. Sahoo, *Advanced Operations Research*, Asian Books Private Limited, New Delhi, 2011.

MPSMA201-4: Boundary Layer Theory and Turbulence

Marks: 50 **Credits: 04** **Total lectures Hours: 50**

Brief history and development of Fluid Mechanics, Fluid properties; Newtonian and non-Newtonian fluids, Different models of non-Newtonian fluids, Nanofluids and its different models, Integral laws for conservation of mass, momentum, angular momentum and energy; Constitutive laws, Differential forms of mass conservation equation, Navier-Stokes Equations; Differential form of Energy equation. Scaling and dimensional analysis, Dynamic Similarity, Laminar and Turbulent flows, Pipe flow, Open channel flow, Boundary layer theory, similarity solutions, high Re flows, creeping flows, steady and unsteady flows, heat transfer coefficients, molecular diffusion in fluids,

mass transfer coefficients, Statistical interpretation of turbulence and its analysis, Spectral analysis and Kolmogorov theory. [35]

Fundamentals of finite difference methods – explicit and implicit schemes; numerical stability and numerical solutions to non-linear ordinary and partial differential equations. [15]

Recommended Books:

1. G. K. Batchelor, *An Introduction to Fluid Dynamics*, Cambridge University Press, 2005.
2. S.W. Yuan, *Foundations of Fluid Mechanics*, Prentice – Hall International, 1970.
3. H. Schlichting, *Boundary Layer Theory*, Springer, 2003.
4. P.K. Kundu and I.M. Cohen, *Fluid Mechanics*, 3rd Ed., Academic Press, 2004.
5. F.S. Sherman, *Viscous Flow*, McGraw Hill International, 1990.
6. K. Muralidhar and G. Biswas, *Advanced Engineering Fluid Mechanics*, 2nd Ed., Alpha Science, 2005.
7. F.M. White, *Viscous Fluid Flow*, McGraw Hill International, 1991.

MPSMA201-5: General Relativity & Relativistic Quantum Mechanics

Marks: 50 **Credits: 04** **Total lectures Hours: 50**

Group – A (General Relativity) (30 Marks)

Principle of equivalence: Mach's principle, [4]

Riemannian geometry: the curvature and stress energy tensors. [6]

The gravitational field equations; geodesics and particle trajectories, Schwarzschild solution. [10]

Experimental tests, basic cosmology, FRW metric; cosmological expansion; cosmic microwave background; helium abundance; anisotropies in the CMBR. [10]

Recommended Books:

1. Bernard F. Schutz, *A First Course in General Relativity*, Cambridge University Press, 2009.
2. Hans Stephani, *General Relativity: An Introduction to the Theory of the Gravitational Field*, Cambridge University Press, 1996.
3. Robert M. Wald, *General Relativity*, University of Chicago Press, 1984.

Group – B (Relativistic quantum mechanics) (20 Marks)

Klein-Gordon equation – plane wave solution, interpretation of K-G equation; Dirac equation – covariant form, charged particle in electromagnetic field, equation of continuity, plane wave solution; Dirac hole theory; Spin of the Dirac particle. [20]

Recommended Books:

1. B. H. Bransden and C. J. Joachain, *Physics of atoms and molecules*, Second Edition, Pearson Education, New Delhi, 2008.
2. Ashok Das, *Lectures On Quantum Mechanics*, Second Edition, World Scientific Publishing Co Pvt Ltd, Singapore, 2012.

MPSMA202: Elective Courses

MPSMA202-1: Riemannian Manifolds and Contact Manifolds

Marks: 50 **Credits: 04** **Total lectures Hours: 50**

Riemannian manifolds: Riemannian manifolds, Riemannian connection, Semi-symmetric connection on Riemannian manifolds, Quarter symmetric connection on Riemannian manifolds, Einstein manifolds and its generalizations, Manifolds of constant curvature and its
generalizations, Some transformations on Riemannian manifolds, Locally symmetric manifolds due to Cartan, Recurrent manifolds and its generalizations, Semi-symmetric manifolds, Pseudo symmetric manifolds, Ricci parallel manifolds, Ricci semi-symmetric manifolds, Ricci pseudo symmetric manifolds, Submanifolds of Riemannian manifolds. [25]

Contact manifolds: Almost contact and contact manifolds, Curvature of contact metric manifolds, k-contact and (k, μ)-contact metric manifolds, Sasakian manifolds, Kenmotsu manifolds, Trans-Sasakian manifolds, Para-Sasakian manifolds, LP-Sasakian manifolds, (LCS)n-manifolds, Sasakian-space-forms, Generalized Sasakian-space-forms, φ-symmetric contact metric manifolds and its generalized classes, Submanifolds of almost contact metric manifolds. [25]

Recommended Books:

1. D. E. Blair, *Riemannian geometry of contact and symplectic manifolds*, Progress in Mathematics, Birkhauser Basel, 2003.
2. U. C. De and A. A. Shaikh, *Complex and contact manifolds*, Narosa Publishing House Pvt. Ltd, 2009.
3. M. P. Do Carmo, *Riemannian Geometry*, Birkhauser, Boston, 1992.
4. U. C. De and A. A. Shaikh, *Differential Geometry of manifolds*, Alpha Science International Ltd. 2007.
5. Luther Pfahler Eisenhart, *Riemannian Geometry*, Princeton University Press, 1997.
6. W. H. Boothby, *An introduction to Differentiable manifolds and Riemannian Geometry*, Academic Press, New York, 1975.
7. John M. Lee, *Riemannian manifolds: An introduction to curvature*, Springer-Verlag, 1997.

MPSMA202-2: Advanced Complex Analysis

Marks: 50 **Credits: 04** **Total lectures Hours: 50**

Review of general theory of entire and meromorphic functions . [5]

Entire functions, $M(r, f)$ and its properties, growth of an entire function, order and type, and their representations in terms of the Taylor coefficients, distribution of zeros. [15]

Meromorphic functions, Definition of $m(r,a)$, $N(r,a)$ and $T(r, f)$. Orders of growth, Order of a meromorphic function. Comparative growth of $\log M(r, f)$ and $T(r, f)$.

Nevanlinna's first fundamental theorem, Cartan's identity and convexity theorems. Picard's Theorem, Nevanlinna's second fundamental theorem, Estimation of $S(r, f)$. Deficiencies of meromorphic functions and their

generalizations, Nevanlinna's Theorem on Deficient Functions, Nevanlinna's five-point Uniqueness Theorem. Milloux Theorem. [20]

Infinite products and related problems. [10]

Recommended Books:

1. A.S.B. Holland, *Introduction to the theory of Entire Functions*, Academic Press, New York, 1973.
2. W. K. Hayman, *Meromorphic Functions*, the Clarendon Press, Oxford, 1964.
3. G. Valiron, *Lectures on the General Theory ofIntegral Functions*, Chelsea Publishing Company, 1949.
4. Lee A. Rubel, *Entire and meromorphic functions*, Springer, 2001.
5. R. P. Boas, *Entire Functions*, Academic Press, 1954.
6. L. Yang, *Value Distribution Theory*, Springer-Verlag, Berlin Heidelberg, New York, 1993.

MPSMA202-3: Algebra -II

Marks: 50 **Credits: 04** **Total lectures Hours: 50**

(By a ring we always mean a commutative ring with 1)

Places, specialization and existence of spaces, the center of place in in subring, the notion of the center of place in algebraic geometry, places and field extensions. [5]

Valuation, Places and Valuation, the rank of a valuation, valuation and field extensions, Ramification theory of general valuation, classical ideal theory and valuation, prime divisors in the field of algebraic functions, Examples of valuation, existence theorem for composite centered valuations, abstract Riemann surface of a field, derived normal model. [15]

Formal power series, graded rings, homogeneous ideal, algebraic varieties in the affine and projective space, homogenous and non-homogenous ideal, [5]

Dimension theory in finite integral domain, special dimension theoretic properties of polynomial rings, normalization theorem, dimension theory in power series ring, extension of the ground field, characteristic functions of graded module and homogenous ideals, chain of syzygies. [10]

Method of associated graded rings, some topological notions, completions, elementary properties of complete modules, Zariski ring, comparison of topology in Noetherian ring, finite extension, Hensel's lemma and applications, characteristic functions, system of parameters, theory of multiplicities, Regular local ring, structure of complete local rings and applications, analytical irreducibilities and analytical normality of normal varieties. [15]

Recommended Books:

1. M. F Atiyah and I.G Macdonald, *An Introduction to Commutative Algebra*, Addision-Wesley Publishing company, 1969.
2. N.H McCoy, *Theory of Rings*, Macmillan, 1964.
3. L. Gillman and M Jerison, *Rings of Continuous Functions*, Springer, New York, 1960.
4. D. S Dummit and R. M . Foote, *Abstract Algebra*, Wiley, Third edition , 2011.

MPSMA202-4: Computational Optimization

Marks: 50 **Credits: 04** **Total lectures Hours: 50**

Advanced inventory models: Fuzzy Inventory models. [10]

Advanced studies of non-traditional optimization techniques: Differential evolution, Ant Colony optimization, Bee colony optimization and applications in Reliability optimization, inventory problems. [20]

Practical by MATLAB, MATHEMATICA, LINGO or by Direct Programming. [20]

Recommended Books:

1. M. Aokie, *Introduction to Optimization Techniques: Fundamentals and Applications of Nonlinear Programming*, The Macmillan Company, 1971.
2. S. Wolfram, *The Mathematica Book*, Wolfram Media, 2003.
3. A. Messac, *Optimization in Practice with MATLAB*, Cambridge, 2015.
4. K. Price, R. M. Stron and J. A. Lampinen, *Differential Evolution: A Practical Approach to Global Optimization*, Springer, 2005.
5. W. Kuo, V. R. Prasad, F. A. Tillman and C. L. Hwang, *Optimal Reliability Design: Fundamentals and Applications*, Cambridge University Press, Cambridge, 2001.
6. A. P. Engelbrecht, *Fundamentals of Computational Swarm Intelligence*, John-Wiley, 2005.

MPSMA202-5: Chaos Theory and Fractals

Marks: 50 **Credits: 04** **Total lectures Hours: 50**

Prerequisite: Ideas on dynamical systems both continuous and discrete systems, fixed points, periodic points, periodic cycles and their stabilities, bifurcations theory and some important maps. [10]

Topological conjugacy, properties of conjugacy, semi-conjugacy relations. [5]

Mathematical theory of chaos: Sensitive dependence on initial condition (SDIC), topological transitivity and mixing, definition of chaotic map. Examples of chaotic maps, ergodic map and ergodic theorem, dynamics of logistic map for $r \geq 4$, symbolic dynamics. [10]

Quantifying chaos: Universal sequence, Feigenbaum number, renormalization group theory and super-stable cycle, Lyapunov exponent and invariant measure. [6]

Sharkovskii's theorem, Li and Yorke theorem, Poincare map, circle map and Smale Horseshoe map. Routes of chaos, universality in chaos. [5]

Fractals: Self-similarity and scaling, self-similar fractals, constructions of self-similar fractals, dimensions of fractals, strange attractors, fractal basin boundary. Applications to fractals in chaotic dynamics and biological systems. [14]

Recommended Books:

1. S. Sternberg, *Dynamical systems*, Diver, 2010.
2. R. L. Devaney, *An Introduction to Chaotic Dynamical Systems*, CRC Press, 2003.
3. G.C. Layek, *An Introduction to Dynamical Systems and Chaos*, Springer, 2015.
4. Edward Ott, *Chaos in Dynamical Systems*, 2nd Ed, CUP, 2002.
5. K.T. Alligood, T.D. Sauer and J.A. Yorke, *Chaos:An Introduction to Dynamical Systems*, Springer, 1997.
6. K. Falconer, *Fractal Geometry, Mathematical Foundationa and Applications*, Wiley, New York, 1990.
7. S. Wiggins, *Introduction to Applied Nonlinear Dynamical Systems and Chaos*, 2nd Ed, Springer, 2003.
8. B. Mandelbrot, *The Frcatal Geometry of Nature*, Freeman & Company, 1982.

MPSMA202-6: Quantum Theory of Scattering & Quantum Field Theory

Marks: 50 **Credits: 04** **Total lectures Hours: 50**

Group-A(Quantum Theory of Scattering) (25 Marks)

Method of partial waves for potential scattering, Integral equation of potential scattering, Scattering by Coulomb potential, Approximate methods for potential scattering. [25]

Group-B (Quantum Field Theory) (25 marks)

Meaning and objective of Quantization of Fields, Types of Fields. [5]

Noether's theorem [3]

(i) Quantization of Scalar Fields, (ii) Quantization of Spinorial (Dirac) Fields, (iii) Quantization of Vector (Electromagnetic) Fields. [5]

ii). The Interaction of Fields, The S-Matrix: Idea and properties, the unitarity of the S-Matrix andthe Optical Theorem. [5]

iii). Iteration solution of the S-Matrix, Ordering theorems and the Introduction to Feynman DiagramTechniques. [7]

Recommended Books:

1. Steven Weinberg, *The Quantum Theory of Fields, Volume 1: Foundations*, 2005.
2. John Polkinghorne, *Quantum Theory: A Very Short Introduction*,Oxford, 2002.
3.Tom Lancaster, *Quantum Field Theory for the Gifted Amateur*, Stephen J. Blundell, Oxford, 2014.

MPSMA203: Term paper

Marks: 50 **Credits: 4**

[Term paper must comprise of either the report on new research works or review of research works on a current research topic done by the candidate under his/her capacity(solely or partially).]